DOCUMENT RESUME

ED 325 307 SE 051 642

TITLE	Mathematics 16, Teacher Resource Manual. Interim--1990.
INSTITUTION	Alberta Dept. of Education, Edmonton. Curriculum Branch.
REPORT NO	ISBN-0-7732-0205-6
PUB DATE	90
NOTE	315p.; For the Program of Studies/Curriculum Guide, see SE 051 641.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC13 Plus Postage.
DESCRIPTORS	Calculators; Cognitive Development; *Cognitive Processes; Computation; Computers; Curriculum Design; Curriculum Development; Decision Making; Foreign Countries; *Learning Strategies; *Mathematics Education; Problem Solving; *Resource Materials; Secondary Education; *Secondary School Mathematics; State Curriculum Guides; Student Attitudes; *Teaching Methods; Technology; Thinking Skills
IDENTIFIERS	*Alberta

ABSTRACT

The Mathematics 16 program provides for the development of essential concepts, skills, and attitudes required for effective computation and problem solving. The program is activity-based, and addresses the need for students to be able to transfer and apply specific mathematical concepts and skills to more generalized situations in everyday life and the world of work. A focus on the use of technology throughout the program assists students to develop the ability to use calculators and computers in performing routine tasks more easily replicated by these technologies. This manual should be used as a practical planning and instructional tool in translating the intentions of the Mathematics 16 program. The themes include earning money, budgeting and banking, skills for the consumer, and math in the workplace. The program emphases and methodology covers problem solving, the use of technology (calculator and computer), computational facility and estimation, situational and concrete approaches, and assessment and evaluation. An annotated list of learning resources, Alberta Consumer and Corporate Affairs Offices, and the regional offices of education are appended. (KR)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

INTEGRATED OCCUPATIONAL PROGRAM

Teacher Resource Manual

ED325307

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
S. Wolodko
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☑ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

MATHEMATICS 16

INTERIM – 1990

SE051642

CURRICULUM

Alberta
EDUCATION

Teacher Resource Manual

Mathematics 16

INTERIM – 1990

NOTE: This publication is a support document. The advice and direction offered is suggested only. Consult the Program of Studies/Curriculum Guide to identify the prescriptive contents of the Mathematics 16 Program.

Alberta. Alberta Education.
Mathematics 16 : teacher resource manual: interim – 1990.

ISBN 0-7732-0205-6

1. Mathematics -- Study and teaching (Secondary). I. Title. II. Integrated Occupational Program.

QA14.A333t 1990 372.7

Copyright © 1990, the Crown in Right of Alberta, as represented by the Minister of Education. Alberta Education, 11160 Jasper Avenue, Edmonton, Alberta, T5K 0L2. All rights reserved. Additional copies may be obtained from the Learning Resources Distributing Centre.

COPYRIGHT

The materials contained in this Teacher Resource Manual have been derived from numerous sources and are designed to provide specific support to selected areas of the curriculum. Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright regulations. If cases are identified where this has not been done, it would be appreciated if Alberta Education could be notified to enable appropriate corrective action to be taken.

ACKNOWLEDGEMENTS

The Mathematics 16 *Teacher Resource Manual* has been developed through the cooperative efforts of many individuals. Each person's contribution is sincerely appreciated. The program has also benefited from the validation of a number of educators across the province. Their comments provided valuable assistance and direction.

ASSOCIATE DIRECTOR: A. A. (Scotty) Day

INTEGRATED OCCUPATIONAL PROGRAM MANAGER: Michael Alpern

PROGRAM MANAGER, MATHEMATICS: Gary Bertrand

INTEGRATED OCCUPATIONAL PROGRAM TEAM:

Jeanne Cooper
Lorraine Crawford
Marilyn Dyck
Linda Elliott
Jan Marvin
Kathy McCabe
Paul McNair

INTEGRATED OCCUPATIONAL PROGRAM REVIEW COMMITTEE:

Stuart Adams – County of Strathcona No. 20
Pat Boon-Anderson – Lac La Biche School Division No. 51
Christopher Harrison – Alberta Vocational Centre, Edmonton
Ed Holt – Conference of Alberta School Superintendents
Alex McCaughan – Alberta Education
Wayne Nixon – Alberta Career Development and Employment
William Smolak – Alberta Teachers' Association
Doug Tarney – Wetaskiwin School District No. 264

EDITING: Elizabeth McCardle

DESKTOP PUBLISHING: Lin Gray
Cheryl Stoochnoff

ILLUSTRATOR: Kelvin Ogden

SECRETARIAL: Rita Flint
Marcie Whitecotton-Carroll

TABLE OF CONTENTS

PAGE

INTRODUCTION

Purpose ... 1
How to Use the Teacher Resource Manual ... 2
Program Planning ... 3
Relationship of Mathematics 16-26 to Mathematics 14-24 ... 5
Scope and Sequence for Mathematics 16-26 ... 7
Learning Resources for Mathematics 16 ... 17
Correlation of Basic Learning Resource to Mathematics 16-26 ... 21

THEMES

A. Earning Money ... 25
    Overview ... 26
    Instructional Strategies ... 31

B. Budgeting and Banking ... 43
    Overview ... 44
    Instructional Strategies ... 49

C. Skills for the Consumer ... 75
    Overview ... 76
    Instructional Strategies ... 81

D. Math in the Workplace ... 109
    Overview ... 110
    Instructional Strategies ... 115

PROGRAM EMPHASES AND METHODOLOGY

PROBLEM SOLVING ... 1

Developing a Desire to Solve Problems ... 2
A Framework for Solving Problems ... 4
Using Algebra in Problem-Solving Situations ... 8
Cooperative Problem Solving ... 13
Monitoring and Evaluating Progress ... 15

USE OF TECHNOLOGY ... 1

The Calculator ... 1
    Entry Procedures ... 2
    Order of Operations ... 3
    Auditory Cueing ... 4
    Rounding Results ... 5
    Finding Remainders ... 5
    Using the Percent Key ... 6
    Using the Memory Keys ... 7
    Checking the Reasonableness of Results ... 8
    Developing Confidence and Speed ... 8

- The Computer 10
  - BASIC Computer Language 11
  - LOGO Computer Language 16

COMPUTATIONAL FACILITY AND ESTIMATION 1

- Developing Computational Process 2
  - Understanding Place Value 2
  - Using Manipulatives 3

- Developing Mental Arithmetic Skills 6
  - Applying Basic Number Skills 6
  - Using Formal Strategies 8

- Developing Strategies for Estimation 10
  - Front-End Estimation 11
  - Rounding 12
  - Compatible Numbers 13
  - Clustering (or Averaging) 14

SITUATIONAL AND CONCRETE APPROACHES 1

- Number Systems and Operations 2
  - Whole Numbers and Decimals 2
  - Integers 6
  - Prime Numbers 7
  - Fractions 10

- Ratio, Proportion and Percent 15

- Geometry and Measurement 20
  - Geometry 20
  - Measurement 23

- Project Work 26
  - Sample Project: Buying a Car 26

ASSESSMENT/EVALUATION 1

- Observation 2
- Interviews 3
- Inventories/Checklists 5
- Anecdotal Records 6
- Written Assignments 7
  - Paper-and-Pencil Tests 7
  - Diagnostic Writing Assignments 9

APPENDICES

Appendix A: Annotated List of Learning Resources 1
Appendix B: Alberta Consumer and Corporate Affairs Offices 6
Appendix C: Regional Offices of Education 7

INTRODUCTION

PURPOSE

This *Teacher Resource Manual* (TRM) has been developed to assist classroom teachers to implement the Integrated Occupational Mathematics 16 Program. Teachers are encouraged to use this manual as a practical planning and instructional tool in translating the intentions of the mathematics program, as outlined in the *Mathematics 16 Program of Studies/Curriculum Guide*, into classroom practice. Although shaded statements or segments within this manual indicate prescriptive content of the Program of Studies, all other advice and direction provided are suggested only and are not legally binding.

This TRM provides:

- further information about the goals and objectives of the mathematics program
- thematic contexts for the delivery of prescribed concepts, skills and attitudes
- suggestions for planning and implementing the program
    - instructional strategies
    - sequenced activities
    - a correlation of strategies and activities to learning resources
- suggestions for relating mathematical instruction to essential life skills and other applications across the curriculum
- suggestions for utilizing community resources in providing instruction in mathematics
- strategies for developing student competence in performing computations, in solving problems, and in using various technologies (i.e., the calculator and computer)
- situational and concrete approaches that give meaning to abstract concepts and skills in mathematics
- strategies for assessing and evaluating student progress.

The binder format was chosen for this manual to enable teachers to add strategies, samples of student work and other activities that have proven effective through experience. During cooperative planning sessions, pages may be easily removed and shared with other Integrated Occupational Program teachers. This exchange will increase awareness of content in other subject areas, thereby providing additional opportunities for students to relate mathematical competencies to applications in everyday life, the occupational courses and other academic disciplines.

8

HOW TO USE THE TEACHER RESOURCE MANUAL

This manual has been organized to provide ready access to both thematic units and suggestions for addressing program emphases and methodology within the Mathematics 16 program.

THEMES

It is recommended that themes provide the vehicle for instruction in Mathematics 16. Four thematic units of instruction that assure coverage of the prescribed concepts, skills and attitudes have been provided in this manual. The themes can be accessed by using the first tab along the side of the manual.

The chart below provides an overview of the content within themes developed for Mathematics 16, and illustrates its relationship to the content within themes developed for Mathematics 26.

	MATHEMATICS 16	MATHEMATICS 26
Theme A **EARNING MONEY**	• FINDING EMPLOYMENT • COMPUTING INCOME	• PAYING INCOME TAX • BUYING AND SELLING
Theme B **BUDGETING AND BANKING**	• USING BANK ACCOUNTS • PREPARING BUDGETS	• BORROWING MONEY • SAVING AND INVESTING
Theme C **SKILLS FOR THE CONSUMER**	• BUYING GOODS AND SERVICES • OPERATING A VEHICLE	• KEEPING A PLACE OF YOUR OWN • BUYING A VEHICLE
Theme D **MATH IN THE WORKPLACE**	• ESTIMATING AND MEASURING • USING SCALE DRAWINGS AND MODELS	• DESIGNING AND BUILDING • MONITORING INVENTORY AND CASH FLOW

Teachers are encouraged to examine each theme and its corresponding activities and learning resources well in advance of instruction. The activities and suggestions provided within each theme are numerous. Advance planning should include a synthesis of effective strategies from the teacher's repertoire of personal experience together with suggestions in this manual considered most appropriate to student needs.

PROGRAM EMPHASES AND METHODOLOGY

Suggestions for addressing program emphases and methodology within the Mathematics 16 program are provided in the five sections that follow the themes. These sections of the manual can also be accessed by using tabs, and are identified as:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Situational and Concrete Approaches
- Assessment/Evaluation.

Each of these sections clarifies the intent of a particular component of instruction, and suggests teaching practices that will assist students to acquire an understanding of prescribed concepts, skills and attitudes. Teachers are encouraged to reference the ideas provided throughout these sections of the manual as they plan learning activities within each theme of the mathematics program.

PROGRAM PLANNING

GENERAL COURSE PLANNING

Themes and their subsequent concepts, skills and attitudes may be sequenced at the teacher's discretion. Program planning should take into consideration the sequential and developmental nature of skills in mathematics as well as students' interests, abilities, and preferred ways of learning. The four themes provided in this manual are intended to be descriptive rather than prescriptive. Teachers may choose to modify these themes in addressing individual student needs.

Through cooperative conferencing, teachers may find that students are required to use certain mathematics-related competencies in other courses before they are learned in mathematics class. Planning and negotiation with teachers of other courses is necessary in establishing an integrated program that places consistent expectations upon the student. (For example, students may benefit from the study of the theme "Math in the Workplace" early in the program as this theme addresses skills frequently demanded of students in their occupational courses.)

Program planning must place emphasis on developing processes that will enable students to cope with the mathematical demands of everyday life. A focus on strategies for "problem solving" will foster students' ability to apply past experience and skills learned in new and unfamiliar situations. A focus on the "use of technology" will assist students to use calculators and computers in performing routine tasks more easily replicated by these technologies. A focus on "computational facility and estimation" will enable students to select and use computational procedures that are appropriate in a variety of practical situations.

The suggestions and ideas provided throughout this manual are numerous, but by no means exhaustive. Be prepared to add, delete and modify activities in adapting a theme to the particular circumstances of the classroom and student.

TIME ALLOCATIONS

As a 3-credit course, Mathematics 16 is designed to be taught through 75 hours of instruction. In meeting student needs, however, local jurisdictions may find it desirable to allocate additional time for instruction in I.O.P. mathematics. (For example, schools may offer this 3-credit course through 125 hours of instruction.)

Time allocations for the delivery of themes are suggested on the page that follows. These suggestions are intended to provide guidance only. Teachers may wish to vary and adjust the allocation of time among individual themes in addressing students' interests, abilities and needs.

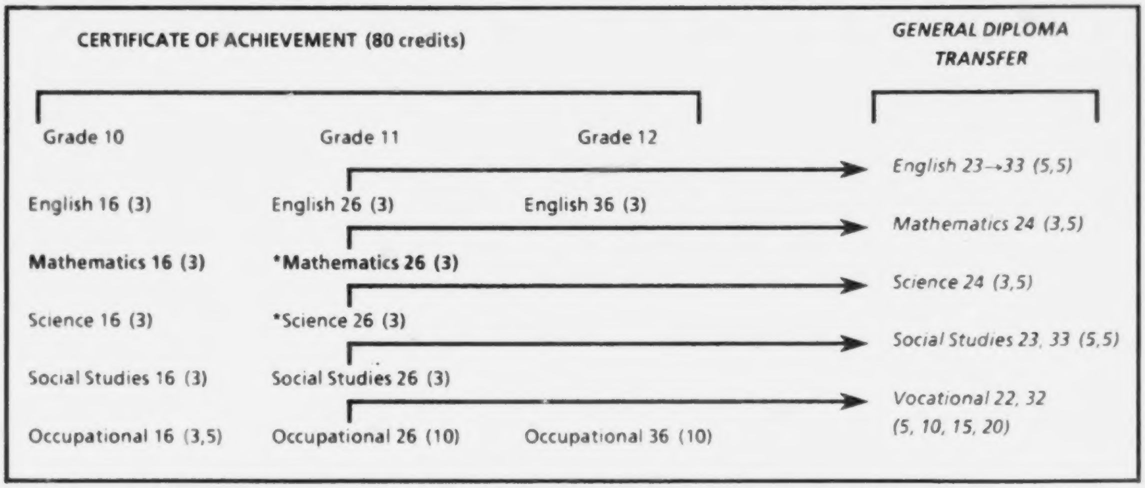
Earning Money	20%
Budgeting and Banking	20%
Skills for the Consumer	20%
Math in the Workplace	20%
Required Time	80%
Elective Time	20%
Total	100%

THEMES	ELECTIVE
EARNING MONEY (20%)	ENRICHMENT/REMEDIATION 20%
BUDGETING AND BANKING (20%)	
SKILLS FOR THE CONSUMER (20%)	
MATH IN THE WORKPLACE (20%)	

RELATIONSHIP OF MATHEMATICS 16-26 TO MATHEMATICS 14-24

RATIONALE

As outlined in the *Guide to Education: Senior High School Handbook* (1990-91), Mathematics 16 is designed to allow students to meet the credit requirements of the Certificate of Achievement. Mathematics 26 is optional, allowing students to further develop their knowledge and skills in mathematics, and facilitates transfer to the General High School Diploma Program. In contrast, the Mathematics 14-24 program is designed to allow students to meet the credit requirements of the General High School Diploma.


CERTIFICATE OF ACHIEVEMENT (80 credits)
GENERAL DIPLOMA TRANSFER
Grade 10
Grade 11
Grade 12
English 23→33 (5,5)
English 16 (3)
English 26 (3)
English 36 (3)
Mathematics 24 (3,5)
Mathematics 16 (3)
*Mathematics 26 (3)
Science 24 (3,5)
Science 16 (3)
*Science 26 (3)
Social Studies 23, 33 (5,5)
Social Studies 16 (3)
Social Studies 26 (3)
Vocational 22, 32 (5, 10, 15, 20)
Occupational 16 (3,5)
Occupational 26 (10)
Occupational 36 (10)

* **Mathematics 26** and Science 26 – These courses are designed to prepare students for possible transfer to the General High School Diploma route. While not required as part of the Certificate of Achievement, I.O.P. students may choose to enrol in these courses and may apply the credits as part of their unspecified course requirements (13 credits).

Both Mathematics 16-26 and Mathematics 14-24 have been developed on the basis of their appropriateness to the developmental levels of students. Both programs, however, provide potential for students to transfer to higher level mathematics courses. Similarly, in both programs, the utility and application of concepts and skills in mathematics provide a focus for instruction.

COURSE DESIGN

Mathematics 16 is heavily contextualized, focusing on thematic organization and situational learning experiences. Mathematics 14 is organized within the context of strands that are characteristic of the discipline. The learning resources selected for use within each course support these methods of instructional design.

INSTRUCTIONAL EMPHASES

Learner objectives stated in the Mathematics 16 *Program of Studies* and the Mathematics 14 *Program of Studies* indicate that these programs parallel one another in the following areas of instructional emphasis:

- the integration of problem solving throughout
- the practical applications made of calculators and computer technologies
- the development and reinforcement of computational skills through practical applications
- the use of a variety of methods of computation (i.e., paper-and-pencil algorithms, mental arithmetic, the calculator, estimation).

CONCEPTS, SKILLS AND ATTITUDES

Concepts, skills and attitudes prescribed in Mathematics 16 and Mathematics 14 are similar within each of the following areas of study:

- problem solving
- number systems and operations/numeration
- ratio, proportion and percent
- measurement.

Concepts, skills and attitudes prescribed in Mathematics 16 are, however, less extensive than those prescribed in Mathematics 14 in each of the following areas of study:

- geometry
- statistics and probability
- graphing
- algebra.

Additional concepts, skills and attitudes prescribed in Mathematics 26 will extend learning in the areas of study noted above.

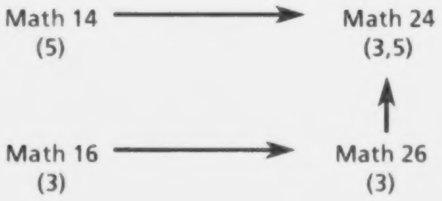
COGNITIVE DEMAND

Cognitive demands in the two programs differ in the level of analysis and abstract thought processes that students are expected to use. Applications in Mathematics 16 generally focus on the transfer of concepts and skills to daily life and work-related situations, whereas many of the applications in Mathematics 14 encourage students to use concepts and skills in more complex situations.

Cognitive demands in these programs will be further influenced by the needs and abilities of individual students.

TRANSFERABILITY

Although content in Mathematics 16 is less extensive than the content in Mathematics 14, the Mathematics 16-26 course sequence will develop concepts, skills and attitudes that facilitate transfer from Mathematics 26 to Mathematics 24. Program sequences and recommended transfer points are illustrated below.



Additional information about transfer routes from the Integrated Occupational Program to the General High School Diploma Program is provided in the *Guide to Education: Senior High School Handbook* (1990-91).

SCOPE AND SEQUENCE FOR MATHEMATICS 16-26

The scope and sequence chart provided on the following pages outlines the mathematical process and skill that is developmentally addressed throughout Mathematics 16 and 26. In recognizing that students differ in the rate at which they acquire mathematical competencies, the chart is intended to assist teachers in assessing present levels of student performance, and in diagnosing individual student strengths and weaknesses.

In using the scope and sequence, it should be noted that:

- although the scope and sequence describes the process and skill that is developmentally addressed throughout the two-year mathematics program, it does not provide a plan for instruction. **Instruction should be sequenced throughout Mathematics 16 and 26 in a manner that will suit individual student needs and growth patterns**

- learning objectives that provide a meaningful **context for instruction** (i.e., problem solving, use of technology, computational facility and estimation) are followed by learning objectives that support the development of process and skill within five **major strands** (i.e., number systems and operations, ratio, proportion and percent, algebra, geometry and measurement, data interpretation and display) established on the basis of tasks frequently demanded of students in everyday life

- effort has been made to arrange process and skill within each strand in a linear sequence according to cognitive demand at each grade level

- the skills are **developmental** through Mathematics 16 and 26 (i.e., the spiral approach). Mathematics 26 provides opportunities for students to reinforce and extend their understanding of skills developed in Mathematics 16 through more extensive applications, and prescribes new skills related to each of the major content strands

- most students will continue to use concrete operational thinking, depending upon personal and tangible experience in order to link ideas. Instruction should generally **begin with an operational understanding of ideas** at the Mathematics 16 level, **and encourage students to extend their thinking to more abstract levels that are cognitively appropriate** at the Mathematics 26 level. (The *Teacher Resource Manuals* for Mathematics 16 and 26 provide for the development of thinking skills through themes that reflect increasing levels of analysis and critical thought)

- **the skills are interdependent and are not meant to be taught in isolation.** Although some skills may be mastered more effectively through discrete instruction, this approach is not advocated as a primary focus of instruction. The thematic structure permits a more holistic view of instruction through the linking of strategies and skills.

Teachers are encouraged to examine a scope and sequence chart for the I.O.P. Junior High School Mathematics Program (Grades 8 and 9). An understanding of the developmental progression of process and skill occurring in junior high school will facilitate program articulation and continuity.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO USE A VARIETY OF STRATEGIES/ SKILLS IN SOLVING PRACTICAL PROBLEMS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.	REFINES AND EXTENDS ABILITY TO USE A VARIETY OF STRATEGIES/ SKILLS IN SOLVING INCREASINGLY DIFFICULT PROBLEMS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.
• Applies strategies/skills that are useful in understanding a problem. e.g. – reads the problem several times – asks questions and refers to other sources to clarify meaning – identifies key words and their meanings – looks for patterns – identifies given and needed information – identifies extraneous information – internalizes the problem by restating in his or her own words, or by visualizing the problem – draws pictures/diagrams – uses concrete manipulatives – interprets pictures/charts/graphs – relates the problem to other problems previously encountered – considers alternative interpretations of the problem.	• Applies strategies/skills that are useful in understanding a problem. → e.g., – determines if there are hidden assumptions that contain information necessary to the solution of the problem.
• Applies strategies/skills that are useful in developing and carrying out a problem-solving plan. e.g. – guesses and checks the result (thus improving the guess) – uses logic or reason – identifies and applies relationships – chooses and sequences the operations needed – sorts and classifies information – applies selected strategies – presents ideas clearly – selects appropriate calculating/measuring devices and methods – acts out or simulates the problem – applies patterns – estimates the answer – documents the process used – works carefully – works in a group situation where ideas are shared – visualizes the problem – makes positive statements (e.g., "I can solve this.") – uses a simpler problem (making an analogy) – identifies factors relevant to the problem – collects and organizes data into diagrams, charts, tables, pictures, graphs or models – experiments through the use of manipulatives – breaks the problem down into smaller parts – formulates an equation – recognizes limits and eliminates possibilities.	• Applies strategies/skills that are useful in developing and carrying out a problem-solving plan. → e.g., – constructs flow charts – works backwards through the problem – examines the problem from varying perspectives/points of view.
• Applies strategies/skills that are useful in reviewing and applying the results of problem solving. e.g. – states an answer to the problem – restates the problem with an answer – explains the answer in oral/written form – determines if the answer is reasonable – discusses with others the process used – suggests other ways of solving the problem – checks the answer – considers the possibility of other answers/solutions – makes and solves similar problems – generalizes the solution and applies the process used to other situations.	• Applies strategies/skills that are useful in reviewing and applying the results of problem solving. → e.g., – creates other routine/non-routine problems that require the use of a similar process.

CONTEXT FOR INSTRUCTION (continued)

	MATHEMATICS 16	MATHEMATICS 26
USE OF TECHNOLOGY	DEMONSTRATES ABILITY TO USE THE HAND-HELD CALCULATOR IN ROUTINE/NON-ROUTINE PROBLEM SITUATIONS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY. • Identifies appropriate and inappropriate uses of the calculator. • Clears the display of the calculator and corrects entry errors. • Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent: – identifies and uses basic functions (i.e., +, −, +, ×, %, =, decimal, clear) – enters numbers in correct sequence for subtraction and division – determines whole number remainders for division – uses subtotals and memory for adding, subtracting, multiplying and dividing – converts fractions to decimal and percent form – generates sets of multiples for a given number – follows order of operations. • Selects from calculator display the number of decimal places appropriate to the context of a calculation/problem. • Uses estimation and mental arithmetic to check the reasonableness of answers obtained on the calculator.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND PROFICIENCY WHEN USING THE HAND-HELD CALCULATOR IN ROUTINE/NON-ROUTINE PROBLEM SITUATIONS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY. → • Identifies and uses additional functions on the calculator (i.e., x^2, y^x, $\sqrt{\ }$).
	DEMONSTRATES AN UNDERSTANDING OF THE BASIC OPERATION, CAPABILITIES, APPLICATIONS AND LIMITATIONS OF COMPUTERS. • Identifies major parts of a computer. • Distinguishes between hardware and software. • Explains how computers get their instructions from a program written to accomplish a specific task. • Gives examples of the applications of computers in performing tasks that require speed, accuracy, repeated operations and the processing of large amounts of data. • Identifies major areas in society where computers are used and the tasks performed by computers in these areas. • Gives examples of tasks that computers cannot accomplish.	DEMONSTRATES AN INCREASED LEVEL OF UNDERSTANDING OF THE OPERATION, CAPABILITIES, APPLICATIONS AND LIMITATIONS OF COMPUTERS. →

CONTEXT FOR INSTRUCTION (continued)

COMPUTATIONAL FACILITY AND ESTIMATION

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO PERFORM COMPUTATIONS AS APPROPRIATE IN DAILY LIFE AND WORK-RELATED SITUATIONS, THROUGH THE USE OF MENTAL ARITHMETIC, PAPER-AND-PENCIL ALGORITHMS, THE CALCULATOR AND ESTIMATION.	DEMONSTRATES INCREASING LEVELS OF COMPETENCE AND ACCURACY WHEN PERFORMING COMPUTATIONS AS APPROPRIATE IN PRACTICAL SITUATIONS, THROUGH THE USE OF MENTAL ARITHMETIC, PAPER-AND-PENCIL ALGORITHMS, THE CALCULATOR AND ESTIMATION.
• Uses mental arithmetic skills that are based upon: – all single-digit operations – sequences of operations – doubling and halving – multiplying and dividing by powers of 10 – applications of the commutative, associative and distributive properties – properties of zero and one – strategies appropriate to the situation (e.g., compensation, computing from left to right)	→
• Uses paper-and-pencil algorithms to perform computations with whole numbers, decimals, fractions, integers and percent within parameters provided in the Scope and Sequence (see "Number Systems and Operations" and "Ratio, Proportion and Percent").	→
• Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent within parameters established by the nature of the problem (see "Use of Technology").	→
• Uses estimation skills that are based upon: – stating the largest and smallest reasonable answer to a problem before solving the problem – predicting whether a computation will result in a larger or smaller number – forecasting an order of magnitude for the result of a computation (e.g., 10's, 100's, 1000's) – predicting the magnitude of the result of a computation through use of strategies appropriate to the situation (e.g., front-end estimation, rounding, clustering, compatible numbers).	→

17

STRANDS

		MATHEMATICS 16	MATHEMATICS 26
NUMBER SYSTEMS AND OPERATIONS	WHOLE NUMBERS	APPLIES WHOLE NUMBER SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. • Identifies place value to one million. • Reads numbers to one million in context. • Write numbers in numeric or word form to one million. • Quantifies and orders numbers to one million. • Identifies even and odd numbers. • Rounds numbers to nearest 10, 100, 1000, 10 000 and 100 000. • Interprets/uses numerical terms (i.e., pair, triplet, decade, dozen, century). • Adds and subtracts whole numbers (limit for paper-and-pencil computation: numbers less than 10 000). • Uses mental arithmetic to determine products when multiplying by multiples of ten. • Multiplies whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100). • Applies divisibility rules for 2, 3, 5 and 10. • Divides whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100). • Determines the lowest common multiple for pairs of numbers less than 10. • Expresses numbers up to 50 as the product of prime factors. • Determines the greatest common factor for pairs of numbers less than 50.	REFINES AND EXTENDS ABILITY TO APPLY WHOLE NUMBER SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. → • Describes the meaning of terms related to the use of exponents (i.e., exponent, base, power, "squared", "cubed"). • Determines the value of a power, using whole number base and exponents of 2 and 3. • Determines the square root of whole numbers, using estimation/square root tables/calculator.
	DECIMALS	APPLIES DECIMAL SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. • Identifies place value to thousandths. • Reads decimals to thousandths in context. • Writes decimals in numeric form to thousandths. • Quantifies and orders numbers to three decimal places in applications. • Rounds decimals to the nearest whole number, tenth and hundredth. • Adds and subtracts decimals (limit for paper-and-pencil computation: numbers to thousandths) • Multiplies decimals (limit for paper-and-pencil computation: numbers that yield products to thousandths, using one- or two-digit multipliers). • Divides decimals (limit for paper-and-pencil computation: numbers having five digits or less by one- or two-digit divisors; dividend may have 0, 1, 2 or 3 decimal places, divisor may be a whole number or decimal). • Uses mental arithmetic to determine products/quotients when multiplying/dividing by multiples of 10.	REFINES AND EXTENDS ABILITY TO APPLY DECIMAL SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. → • Recognizes place value beyond the thousandths place in work-related situations (e.g., reading the micrometer). • Rounds to the nearest thousandth/ten thousandth as required in work-related situations.

STRANDS (continued)

		MATHEMATICS 16	MATHEMATICS 26
NUMBER SYSTEMS AND OPERATIONS (cont'd)	FRACTIONS	DEMONSTRATES ABILITY TO USE BASIC FRACTION SKILLS IN PRACTICAL PROBLEM SITUATIONS (WITH EMPHASIS ON FRACTIONS HAVING DENOMINATORS OF 2, 3, 4, 5, 8, 10 AND 16). • Illustrates a fraction as part of a whole, part of a set, or a point on a number line. • Describes proper/improper fractions and mixed numbers through the use of objects, pictures and diagrams. • Using a number line, illustrates the relationship between whole numbers, decimals and fractions. • Compares and orders fractions in applications. • Identifies and determines equivalent fractions (including proper and improper fractions, mixed numbers). • Identifies and expresses fractions in basic form. • Converts proper/improper/mixed fractions to decimals using a calculator. • Recalls decimal equivalents for one-half, thirds, quarters, fifths and tenths. • Determines common denominators for fractions having denominators of 2, 3, 4, 5, 8, 10 and 16. • Adds/subtracts fractions and mixed numbers having like/unlike denominators that are within the parameters stated above. • Multiplies/divides proper fractions and mixed numbers by whole numbers. • Calculates a fractional part of an amount in applications.	REFINES AND EXTENDS ABILITY TO USE BASIC FRACTION SKILLS IN PRACTICAL PROBLEM SITUATIONS (WITH EMPHASIS ON FRACTIONS HAVING DENOMINATORS OF 2, 3, 4, 5, 8, 10 AND 16). → • Multiplies/divides proper fractions and mixed numbers by whole numbers/proper fractions.
	INTEGERS	DEMONSTRATES ABILITY TO USE INTEGER SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS. • Identifies situations that require the use of integers. • Uses vocabulary related to integers (i.e., positive, negative, plus, minus, above, below, gain, loss). • Places integers on the number line. • Compares and orders integers in practical applications. • Adds pairs/groups of one- and two-digit integers.	REFINES AND EXTENDS ABILITY TO USE INTEGER SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS. → • Adds/subtracts pairs of integers containing three digits or less. • Multiplies/divides pairs of one- and two-digit integers (where multiplier and divisor are positive). • Describes the use of integers in locating points on the coordinate plane.

STRANDS (continued)

	MATHEMATICS 16	MATHEMATICS 26
RATIO, PROPORTION AND PERCENT	DEMONSTRATES ABILITY TO USE RATIO, PROPORTION AND PERCENT SKILLS IN EVERYDAY APPLICATIONS AND PROBLEM-SOLVING SITUATIONS. • Compares two quantities in the same unit by writing ratios in the form a/b, a:b and a to b. • Generates equivalent ratios. • Calculates rates/unit rates by writing ratios that involve numbers with different units (e.g., 90 km/2h, 3 items for $1.00). • Identifies proportions as statements about equivalent ratios. • Writes proportions that describe practical problem situations. • Calculates the unknown value in a proportion. • Expresses whole number percents as ratios/decimals. • Expresses ratios as decimals and percents (i.e., $\frac{a}{b} = \frac{?}{100}$ where b = 2, 4, 5, 10, 20, 25 or 50.) • Recalls fraction, decimal and percent equivalents for one-half, quarters and tenths. • Expresses one-, two- and three-place decimals as percents. • Determines other fraction, decimal and percent equivalents through the use of the calculator. • Calculates/estimates a percent of a number.	REFINES AND EXTENDS ABILITY TO USE RATIO, PROPORTION AND PERCENT SKILLS IN EVERYDAY APPLICATIONS AND PROBLEM-SOLVING SITUATIONS. → • Using a calculator, converts fractional percents to fraction and decimal forms (e.g., 12 1/2% = $\frac{1}{8}$ = 0.125). → • Calculates/estimates what percent one number is of another. • Calculates/estimates any one of the missing elements in practical problems involving application of percent (i.e., determines percentage, rate or base).
ALGEBRA	DEMONSTRATES ABILITY TO USE ALGEBRAIC SKILLS WITHIN THE CONTEXT OF EVERYDAY AND WORK-RELATED SITUATIONS. • Distinguishes between the use of variables and constants in practical situations. • Uses variables to write mathematical expressions/equations that describe familiar situations. • Evaluates expressions for given whole number values of the variable. • Uses estimation, guess/check, concrete manipulatives and formal strategies to find whole number solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$). • Applies equation-solving skills to practical problem situations. • Interprets formulas as word statements. • Uses formulas and substitution to solve problems in practical situations (e.g., if $P = 2L + 2w$, find P when $L = 10$ and $w = 8$).	DEMONSTRATES ABILITY TO USE ALGEBRAIC SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS THAT REQUIRE INCREASING LEVELS OF ANALYSIS AND CRITICAL THOUGHT. → • Evaluates expressions for given whole number/decimal values of the variable. • Uses estimation, guess/check, concrete manipulatives and formal strategies to find whole number/decimal solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$). • Applies equation-solving strategies/skills to practical situations involving whole numbers/decimals. • Uses substitution and equation-solving strategies to find the missing element in a formula (e.g., if $P = 2L + 2w$, find L when $P = 16$ and $w = 3$).

		MATHEMATICS 16	MATHEMATICS 26
GEOMETRY AND MEASUREMENT	GEOMETRY	DEMONSTRATES ABILITY TO APPLY GEOMETRY SKILLS IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies line relationships (i.e., horizontal, vertical, parallel, perpendicular and intersecting). • Identifies/constructs basic two-dimensional geometric figures (i.e., rectangle, square, parallelogram, triangle). • Classifies triangles and quadrilaterals by examining measures of sides and measures of angles. • Identifies the relationship between the radius and diameter of a circle. • Constructs circles, given either radius or diameter. • Identifies similar and congruent geometric figures. • Identifies/sketches/constructs models of basic three-dimensional shapes (i.e., rectangular solid, cube, cylinder, cone). • Constructs geometric figures/designs, using a variety of construction tools (i.e., straightedge, ruler, compass, protractor, computer).	DEMONSTRATES INCREASING CONFIDENCE AND ABILITY TO APPLY GEOMETRY SKILLS IN DAILY LIFE AND WORK-RELATED SITUATIONS. → • Identifies/classifies polygons according to the number of sides. • Constructs regular polygons (i.e., triangle, square, hexagon, octagon). → • Describes the terminology associated with the coordinate system (i.e., origin, axis, quadrant, coordinates, ordered pair). • Plots a point on the coordinate plane when given its coordinates, and identifies the coordinates for a point graphed in the coordinate plane.
	LENGTH	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE LENGTH IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of length (i.e., mm, cm, m, km). • Estimates/measures length, using metric units and tools appropriate to the situation. • Converts among SI units of length as required in applications. • Estimates/measures/calculates the perimeter of figures bounded by line segments using units and tools appropriate to the situation. • Identifies the value of π as the ratio of the circumference of a circle to its diameter. • Develops/applies strategies for determining the circumference of a circle.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING LENGTH IN DAILY LIFE AND WORK-RELATED SITUATIONS. → • Develops/applies strategies for determining the perimeter of any regular polygon. • Describes the Pythagorean relationship among sides of a right triangle, and applies the relationship to practical situations.
	AREA	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE AREA IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of area (i.e., cm^2, m^2) and their application in practical situations. • Approximates the area of two-dimensional geometric figures using a square grid. • Estimates/calculates area of the rectangle, square, triangle and circle, using units, tools and strategies appropriate to the situation.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING AREA IN DAILY LIFE AND WORK-RELATED SITUATIONS. → • Develops/applies appropriate strategies for determining the surface area of rectangular solids and cylinders.

STRANDS (continued)

		MATHEMATICS 16	MATHEMATICS 26
GEOMETRY AND MEASUREMENT (cont'd)	VOLUME	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE VOLUME IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of volume (i.e., cm³, m³) and their application in practical situations. • Approximates the volume of rectangular solids using cubes. • Estimates/calculates the volume of rectangular solids and cubes, using units, tools and strategies appropriate to the situation.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING VOLUME IN DAILY LIFE AND WORK-RELATED SITUATIONS. → • Identifies the equivalence between cubic centimetres and millilitres. • Develops/applies appropriate strategies for determining the volume of right cylinders.
	CAPACITY	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE CAPACITY IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of capacity (i.e., mL, L) and their application in practical situations. • Estimates and measures capacity, using metric units and tools appropriate to the situation. • Converts between mL and L.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING CAPACITY IN DAILY LIFE AND WORK-RELATED SITUATIONS. →
	MASS	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE MASS IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of mass (i.e., g, kg, t) and their application in practical situations. • Estimates and measures mass, using metric units and tools appropriate to the situation. • Converts between g and kg and between kg and t.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING MASS IN DAILY LIFE AND WORK-RELATED SITUATIONS. →
	TIME	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE TIME IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Uses a calendar, recognizing the relationship between days, weeks, months and years. • Uses National Standards for numeric dating. • Estimates/measures/records time on the 12-hour and 24-hour clocks, using traditional and digital time pieces. • Determines the interval between a pair of times. • Determines the finishing time, given the starting time and duration.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILLS WHEN ESTIMATING AND MEASURING TIME IN DAILY LIFE AND WORK-RELATED SITUATIONS. →
	TEMPERATURE		DEMONSTRATES ABILITY TO ESTIMATE AND MEASURE TEMPERATURE IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Estimates/measures temperature on the Celsius scale. • Recalls important temperatures on the Celsius scale (i.e., boiling/freezing point of water, normal room/body temperature). • Determines temperature change, including changes from below zero to above zero and vice versa.
	ANGLE	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE ANGLES IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies an angle and the degree as a unit of measure. • Identifies/estimates angles of 30°, 45°, 60°, 90°, 180° and 360°. • Uses a protractor to measure and draw angles from 0° to 180°.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING ANGLES IN DAILY LIFE AND WORK-RELATED SITUATIONS • Uses a protractor to measure and draw angles from 0° to 360°. • Applies knowledge of the sum of the angles of a triangle in practical situations.

STRANDS (continued)

	MATHEMATICS 16	MATHEMATICS 26
DATA INTERPRETATION AND DISPLAY	COLLECTS, ORGANIZES, DISPLAYS AND ANALYZES NUMERICAL DATA IN ORDER TO DRAW VALID INFERENCES AND MAKE INFORMED DECISIONS IN PRACTICAL EVERYDAY SITUATIONS. • Distinguishes between a survey and census, also between a sample and population, identifying potential biases that may occur in surveys and samples. • Interprets and analyzes data presented in tables/charts and bar/line/picture/circle graphs. • Displays data in the form of tables/charts and bar/line/picture/circle graphs. • Identifies the characteristics of a misleading graphical display. • Determines/calculates appropriate measures of central tendency in practical situations (i.e., mean, median, mode, range). • Conducts a survey or poll, using appropriate methods of gathering, organizing, presenting and analyzing data: – selects a suitable sample from a given population – collects and organizes data using tally sheets/frequency tables – displays data in the form of bar/line/picture/circle graphs – makes inferences/generalizations about the population from which a sample was taken.	COLLECTS, ORGANIZES, DISPLAYS AND ANALYZES NUMERICAL DATA IN ORDER TO DRAW VALID INFERENCES AND MAKE INFORMED DECISIONS IN PRACTICAL SITUATIONS THAT REQUIRE INCREASING LEVELS OF ANALYSIS AND CRITICAL THOUGHT. → • Explains the meaning of "probability" and gives examples of its use in today's society. • Uses counting techniques such as tree diagrams to find the number of possible outcomes of an experiment. • Expresses the probability of the occurrence of a single event related to practical experience or a simple experiment.

LEARNING RESOURCES FOR MATHEMATICS 16

The following learning resources address goals and objectives of the Mathematics 16 program. The abbreviations for some resources that have been referenced throughout the TRM are noted in brackets. A description of each learning resource is provided in Appendix A.

STUDENT RESOURCES

BASIC LEARNING RESOURCE

The learning resource listed below addresses the majority of the goals and learning objectives identified in the *Program of Studies/Curriculum Guide*.

> Carli, Enzo et al. *Consumer and Career Mathematics, Second Canadian Edition* (CCM). Agincourt, Ontario: Gage Educational Publishing Company, 1991.

SUPPORT LEARNING RESOURCE

The learning resource listed below assists in addressing some of the learning objectives identified in the *Program of Studies/Curriculum Guide*. Multiple copies (but not necessarily class sets) may be desired.

> Cutting, Robert, et al. *Math You Need, Book 2* (MYN). Scarborough, Ontario: Nelson Canada, A Division of International Thomson Limited, 1982.

OTHER LEARNING RESOURCES

Other learning resources that may be useful in implementing the Mathematics 16 program are identified below. These resources have not been evaluated by Alberta Education, and their listing is not to be construed as an explicit or implicit departmental approval for use. These titles are provided as a service only to assist local jurisdictions in identifying potentially useful learning resources. The responsibility for evaluating these resources prior to selection rests with the local jurisdiction.

> *A Student Guide to Banking: Students' Portfolio*. Toronto, Ontario: Canadian Imperial Bank of Commerce.

> Alberta Consumer and Corporate Affairs

Booklets: *Consumer Complaints*
Moving Out
Taking Charge of Your Money
"Consumer Talk" Information Sheets:
Returns
Shop Around
Shopping at Home

Dlabay, Les. *Decisions, Making Personal Economic Choices*. St. Paul, Minnesota: EMC Publishing Corporation, 1985.

Elrick, Thomas. *Forms in Your Life: A Student Workbook and Guide to Everyday Forms*. Toronto, Ontario: Globe/Modern Curriculum Press, 1981.

Lennox, Art. *Banking, Budgeting and Employment*. Phoenix, New York: Frank E. Richards Publishing Company Incorporated, 1984.

Lyng, Merwin, et al. *Applied Mathematics*. Markham, Ontario: Houghton Mifflin Canada Limited, 1989.

Wool, John. *Useful Arithmetic, Volume 1*. Phoenix, New York: Richards Publishing Company, 1987.

Wool, John. *Useful Arithmetic, Volume 2*. Phoenix, New York: Richards Publishing Company, 1988.

TEACHER RESOURCES

The teacher resources listed below assist in addressing major goals and learning objectives identified in the *Program of Studies/Curriculum Guide*.

Alberta Education: *Problem-Solving Challenge for Mathematics*. Edmonton, Alberta: Alberta Education, 1985.

Alberta Education: *Problem Solving in Mathematics: Focus for the Future*. Edmonton, Alberta: Alberta Education, 1987.

Charles, R., et al. *How to Evaluate Progress in Problem Solving*. Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Easterday, K., et al. *Activities for Junior High School and Middle School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1981.

Hope, J., et al. *Mental Math in Junior High*. Palo Alto, California: Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1989.

TECHNOLOGY AND MEDIA

There is a growing collection of technology and media resources (e.g., films, videos, computer software, kits, pamphlets) that support the development of specific learning objectives within the mathematics program. Some of these resources have been identified below. Teachers are encouraged to contact local media services, libraries and government agencies in obtaining these and other resources considered appropriate to the mathematics program.

SUPPORT LEARNING RESOURCES

The computer software programs listed below assist in addressing some of the learning objectives identified in the *Program of Studies/Curriculum Guide*.

Math Strategies: Problem Solving. Willowdale, Ontario: Science Research Associates (Canada) Limited, 1983.

Money Manager. Washington, D.C.: Computer Age Education Incorporated, 1984.

OTHER LEARNING RESOURCES

Other computer software programs that may be useful in implementing the Mathematics 16 program are identified below. These resources have not been evaluated by Alberta Education, and their listing is not to be construed as an explicit or implicit departmental approval for use. The responsibility for evaluating these resources prior to selection rests with the local jurisdiction.

Managing Lifestyles: Survival Math Skills. Pleasantville, New York: Sunburst Communications Incorporated, 1984.

Spreadsheets for Students. Mississauga, Ontario: Copp Clark Pitman Limited, 1985.

SRA Computer Drill and Instruction, Mathematics, Level D. Willowdale, Ontario: Science Research Associates (Canada) Limited, 1981-83.

An annotated list of additional computer software available in mathematics is provided in Alberta Education's catalogue of *Computer Courseware Evaluations* (Curriculum Support Branch, Student Programs and Evaluation Division, Alberta Education, 1986) and annual supplements.

The video programs listed below also assist in addressing some of the learning objectives identified in the *Program of Studies/Curriculum Guide*. These programs are available through ACCESS NETWORK and/or Regional Resource Libraries.

Mathways. Kinetic, 1980.

Math Wise. Bloomington, Indiana: Agency for Instructional Television, 1981.

ACCESS NETWORK

ACCESS offers a variety of resources and services to teachers. For a nominal dubbing and tape fee, teachers may have ACCESS audio and video library tapes copied. ACCESS also offers a service called "Night Owl Dubbing". This allows educators to tape late night educational programs directly from their own televisions.

ACCESS publishes both an *Audio-Visual Catalogue* and a comprehensive schedule of programming, available on request.

For additional information, contact ACCESS NETWORK, Media Resource Centre, 295 Midpark Way S.E., Calgary, Alberta, T2X 2A8 (from outside of Calgary, telephone toll free, 1-800-352-8293; in Calgary, telephone 256-1100).

REGIONAL RESOURCE LIBRARIES

Films and videos are available for loan through the five centres listed below. In some instances, computer software is also loaned. Catalogues of holdings are available upon request.

Zone I
Zone One Regional Film Centre
P.O. Box 6536 / 10020 – 101 Street
Peace River, Alberta
T8S 1S3
Telephone: 624-3187

Zones II and III
Central Alberta Media Service (CAMS)
2017 Brentwood Boulevard
Sherwood Park, Alberta
T8A 0X2
Telephone: 464-5540 / 467-8896

Zone IV
Alberta Central Regional Education Services (ACRES)
County of Lacombe No. 14
Box 3220 / 5140 – 49 Street
Lacombe, Alberta
T0C 1S0
Telephone: 782-5730

Zone V
South Central Alberta Film Federation (SCAFF)
Westmount School
Box 90 / Wheatland Trail
Strathmore, Alberta
T0J 2H0
Telephone: 934-5028

Zone VI
Southern Alberta Regional Film Centre (SARFC)
c/o McNally School
P.O. Box 845
Lethbridge, Alberta
T1J 3Z8
Telephone: 320-7807

CORRELATION OF BASIC LEARNING RESOURCE TO MATHEMATICS 16-26

A correlation of *Consumer and Career Mathematics, Second Canadian Edition* (1991) to Mathematics 16 and Mathematics 26 is provided below. Topics in the textbook identified as core provide focused development of concepts and skills prescribed in the *Program of Studies*, and can be used as required to support skills-based instruction. Topics identified as extension provide thematic contexts for developing and reinforcing prescribed content, and can be used as appropriate in assisting students to extend, transfer and apply prescribed content to more generalized situations in everyday life and the world of work. Suggestions for integrating core and extension components of the textbook into thematic units of instruction are provided in an ensuing section of this manual (see "Themes").

The correlation illustrates how students will reinforce and extend their understanding of concepts and skills developed in Mathematics 16 through new thematic contexts and more extensive applications in Mathematics 26. **Although authorization of *Consumer and Career Mathematics* as a basic learning resource for Mathematics 26 is pending, an understanding of how this learning resource correlates to the Mathematics 16-26 course sequence will facilitate program planning and continuity.**

CONSUMER AND CAREER MATHEMATICS, The Second Canadian Edition (1991)	MATH 16		MATH 26	
	Core	Extension	Core	Extension
UNIT 1: MATHEMATICAL SKILLS				
Chapter 1: Whole Numbers, Decimals and Fractions				
Rounding Whole Numbers and Decimals	p. 4			
Adding and Subtracting Whole Numbers/Decimals	p. 6			
Multiplying Whole Numbers and Decimals	p. 8			
Estimation	p. 10			
Dividing Whole Numbers and Decimals	p. 12			
Comparing and Renaming Fractions/Mixed Numbers	p. 14			
Multiplying and Dividing Fractions/Mixed Numbers			p. 16	
Adding and Subtracting Fractions/Mixed Numbers			p. 18	
Chapter 2: Equations, Proportions and Percent				
Addition and Subtraction Equations			p. 24	
Multiplication and Division Equations			p. 26	
Two-Step Equations			p. 28	
Ratio and Proportion	p. 30			
Writing Decimals and Fractions as Percents	p. 32			
Writing Percents as Decimals and Fractions	p. 34			
Finding a Percent of a Number	p. 36			
Finding What Percent One Number Is of Another			p. 38	
Finding a Number When a Percent of It Is Known			p. 40	
Percent Increase and Decrease			p. 42	
Estimating Answers to Percent Problems			p. 44	
Chapter 3: Measurement and Geometry				
Metric Units of Length	p. 50			
Metric Units of Capacity and Mass	p. 52			
Temperature and Time	p. 54		p. 54	
Computing with Measures	p. 56			
Lines and Angles	p. 58			
Classifying Polygons			p. 60	
Perimeter and Area of a Polygon	p. 62			
Circumference and Areas of a Circle	p. 64			
Volume of a Rectangular Solid	p. 66			

CONSUMER AND CAREER MATHEMATICS, The Second Canadian Edition (1991)	MATH 16		MATH 26	
	Core	Extension	Core	Extension
UNIT 2: PROBLEM SOLVING, PROBABILITY, STATISTICS				
Chapter 4: Problem-Solving Skills and Strategies				
A Five-Step Method	p. 76		p. 76	
Solving Multiple-Step Problems	p. 78		p. 78	
Guess and Check	p. 80		p. 80	
Use a Physical Model	p. 82		p. 82	
Draw a Diagram	p. 84		p. 84	
List All the Possibilities	p. 86		p. 86	
Look for a Pattern	p. 88		p. 88	
Solve a Related but Simpler Problem	p. 90		p. 90	
Chapter 5: Probability				
Tree Diagrams and the Counting Principle			p. 96	
Permutations			p. 98	
Probability			p. 100	
Determining Odds			p. 102	
Computing Probability			p. 104	
Making Predictions			p. 106	
Chapter 6: Statistics				
Organizing Data	p. 112			
Bar Graphs and Histograms	p. 114			
Broken-Line Graphs	p. 116			
Circle Graphs	p. 118			
Measures of Central Tendency	p. 120			
Stem-and-Leaf Plots	p. 122			
Statistics in Media	p. 124			
UNIT 3: INCOME, PURCHASING AND BUDGETING				
Chapter 7: Income				
Hourly Rate and Overtime Rate		p. 134		
Hourly Rate Plus Tips		p. 136		
Straight Commission		p. 138		
Bookkeeper		p. 140		
Graduated Commission		p. 142		
Payroll Deductions: Income Tax		p. 144		
Payroll Deductions: Social Insurance		p. 146		
Net Pay		p. 148		
Chapter 8: Buying Food and Goods				
Nutritionist		p. 156		
Calorie Usage		p. 158		
Grocery Shopping		p. 160		
Comparing Meal Costs		p. 164		
Catalog Buying		p. 166		
Seasonal Sales		p. 168		
Rental Clerk		p. 170		
Chapter 9: Budgeting				
Analyzing Spending Habits		p. 178		
Budgeting Variable Expenses		p. 182		
Making a Personal Budget		p. 184		
Adjusting a Budget		p. 188		
Economist		p. 190		

CONSUMER AND CAREER MATHEMATICS, The Second Canadian Edition (1991)

	MATH 16		MATH 26	
	Core	Extension	Core	Extension
UNIT 4: BANKING AND CREDIT				
Chapter 10: Personal Banking				
Deposit Slips		p. 202		
Checks and Check Stubs		p. 204		
Check Registers		p. 206		
Bank Statements		p. 208		
Reconciling a Bank Statement		p. 210		
Simple Interest		p. 214		
Compound Interest in Savings Accounts		p. 216		
Compound Interest Tables		p. 218		
Chapter 11: Consumer Credit				
Promissory Notes				p. 226
Credit Card Finance Charges				p. 228
Credit Counsellor				p. 230
Minimum Payments on Charge Accounts				p. 232
Level-Payment Loans				p. 234
Installment Buying				p. 236
Comparing Credit Plans				p. 238
Home Equity Loans				p. 240
UNIT 5: TRANSPORTATION				
Chapter 12: Buying a Car				
Finding the Total Cost of Buying a Used Car				p. 252
New Car Sticker Price				p. 254
Estimating the Amount to Offer for a New Car				p. 256
Shopping for a New Car				p. 258
Financing a Car				p. 260
Automobile Salesperson				p. 264
Chapter 13: Automobile Operating Expenses				
Finding Gasoline Costs		p. 272		
Automobile Depreciation		p. 274		
Automobile Mechanic		p. 276		
Automobile Liability Insurance		p. 280		
Automobile Collision and Comprehensive Insurance		p. 284		
Annual Automobile Expenses		p. 286		
Alternatives to Owning a Car		p. 288		
Chapter 14: Travel				
Reading a Distance Chart				
Finding Distance and Travel Time				
Expenses on the Road				
Renting a Car				
Travel Agent				

CONSUMER AND CAREER MATHEMATICS, The Second Canadian Edition (1991)

	MATH 16		MATH 26	
	Core	Extension	Core	Extension
UNIT 6: HOUSING				
Chapter 15: Renting and Decorating a Home				
Estimating the Amount to Spend for Rent				p. 316
Selecting a Place to Rent				p. 318
Personal Property Insurance				p. 320
Estimating Electricity Costs				p. 322
Installing Floor Tiles				p. 324
Painting an Apartment				p. 326
Interior Designer				p. 328
Chapter 16: Buying a Home				
Estimating the Amount to Borrow for a Home				
Down Payment and Monthly Payment				
Adjustable-Rate Mortgages				
Interest on a Fixed-Rate Mortgage Loan				
Principal and Interest in a Monthly Payment				
Closing Costs				
Homeowner's Insurance				
Real Estate Agent				
Chapter 17: Building a Home				
Surveyor				
Cost of Building a House				
Cost of Installing a Driveway				
Bricklayer				
UNIT 7: TAXES, INSURANCE AND INVESTMENTS				
Chapter 18: Income Tax				
Income Tax and the T4 Slip				p. 378
Individual Income Tax Return				p. 380
Finding Taxable Income				p. 384
Finding Total Non-Refundable Tax Credits				p. 388
Tax Refund or Balance Due				p. 392
Provincial Income Tax				p. 396
Income Tax Consultant				p. 398
Chapter 19: Health and Life Insurance				
Health Insurance				
Term Life Insurance				
Straight Life Insurance				
Disability Income Insurance				
Insurance Agent				
Choosing Insurance and Savings Plans				
Chapter 20: Investments				
Canada Savings Bonds				p. 426
Guaranteed Investment Certificates				p. 428
Common Stock				p. 432
Investment Counsellor				p. 436

THEME A
EARNING MONEY

SUB-THEMES

- FINDING EMPLOYMENT
- COMPUTING INCOME

RATIONALE

This theme provides opportunities for students to:

- develop and apply appropriate job search skills
- calculate the wages, deductions and earnings associated with a variety of jobs available in the local community.

Employment skills developed within this theme are intended to establish a foundation for further investigations in Mathematics 26, where students will develop skills that are used in filing an income tax return and monitoring profit/loss in business and trade-related situations.

Investigations will focus attention on potential employment opportunities that exist in the local community, and a strategy for evaluating various types of employment on the basis of personal characteristics and job-related factors. Students will also consolidate and reinforce computational skills involving the use of whole numbers, decimals, fractions and percent as they investigate the earnings associated with different types of employment. Calculators should be used on a regular basis in performing routine wage calculations.

Cooperative planning among teachers will ensure that activities within this theme complement job search activities that may be undertaken by students in other subject areas. The community will provide many opportunities for students to develop concepts and skills through the first-hand investigation of potential employment opportunities and practical experiences related to part-time jobs already held by students.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed in this theme can be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Assessment/Evaluation.

THEMATIC OBJECTIVES

FINDING EMPLOYMENT

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- identify personal characteristics that need to be considered when searching for employment (e.g., interests, aptitudes, aspirations)
- use a variety of strategies to explore current employment opportunities in the community (e.g., newspapers, employment centres, interviews, job shadowing, work experience)
- compare the features of jobs that are available in the community (e.g., hours of work, remuneration, working conditions, skills required)
- evaluate the appropriateness of various employment opportunities by considering personal interests/aptitudes/aspirations and job features
- demonstrate an understanding that making a job application may involve completing a number of tasks (e.g., writing a letter of application, completing an application form, attending an interview, writing a pre-employment test, providing a social insurance number).

COMPUTING INCOME

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- demonstrate an understanding of different kinds of wages (e.g., hourly, weekly, bi-weekly, monthly, annual, commissions, contract work)
- explain the meaning of "minimum wage" and identify its current value
- calculate an overtime wage, based upon time-and-a-half or double time
- determine the number of hours worked from personal records or from a time card
- distinguish between gross earnings and net earnings
- calculate gross earnings (e.g., daily, weekly, bi-weekly, monthly, annual), given the time worked and the rate of pay
- calculate earnings based upon commission or contract work
- identify compulsory deductions from earnings (e.g., income tax, Canada Pension, Unemployment Insurance)
- identify voluntary deductions from earnings (e.g., retirement pensions, union dues, medical insurance, disability insurance)
- calculate net earnings, given gross earnings and deductions.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

Students will be expected to:

- gather information about current employment opportunities in the local community
- evaluate the appropriateness of various employment opportunities on the basis of personal interests/aptitudes/aspirations and job features
- maintain a personal record of time worked, and determine gross earnings, deductions and net earnings in real or simulated situations.

USE OF TECHNOLOGY

Students will be expected to:

- use a calculator to determine overtime wage, gross earnings, commissions, deductions and net earnings
- work independently with prepared computer software as required to develop/maintain mathematical concepts and skills
- develop an understanding of the use of computer technology in preparing/maintaining personal records of earnings.

COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform computations with the calculator on a regular basis
- round as appropriate during calculations (e.g., money to the nearest cent, hours worked to the nearest quarter hour)
- use mental arithmetic where appropriate to expedite solutions to quantitative problems (e.g., in determining time-and-a-half pay for overtime, in determining total hours worked)
- estimate potential earnings, deductions and net pay in real or simulated situations
- use estimation to check the reasonableness of calculations and solutions.

SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

Number Systems and Operations	Ratio, Proportion and Percent	Algebra	Geometry and Measurement	Data Interpretation and Display

NUMBER SYSTEMS AND OPERATIONS

Students will be expected to:

- read, write and order whole numbers, decimals and fractions in applications
- identify place value from 0.01 to 1 000 000
- round whole numbers and decimals as required
- perform basic operations with whole numbers and decimals
- perform a sequence of operations in correct order
- add and subtract fractions
- determine decimal equivalents for commonly used fractions.

RATIO, PROPORTION AND PERCENT

Students will be expected to:

- express whole number percents as ratios and decimals
- estimate and calculate a percent of a number.

ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions/equations that describe practical situations
- apply equation-solving strategies to practical situations
- interpret formulas as word statements
- use substitution and equation-solving strategies to find the missing element in a formula.

GEOMETRY AND MEASUREMENT

Students will be expected to:

- use a calendar, recognizing the relationship between days, weeks, months and a year
- use National Standards for numeric dating
- estimate, measure and record time on a 12-hour and 24-hour clock
- determine the interval between a pair of times
- determine the finishing time, given the starting time and duration.

DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- read and interpret tables, charts and graphs that provide information about various kinds of employment
- use tables/charts that indicate weekly or monthly deductions from earnings (e.g., tax deductions, Canada Pension Plan contributions, Unemployment Insurance premiums)
- interpret and/or calculate appropriate measures of central tendency in practical situations.

INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with job search activities that are undertaken by students in other subject areas. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

ENGLISH

Students may:

- write a letter of application for employment
- read, interpret and complete various employment forms
  - application for S.I.N.
  - employment registration with Manpower and Immigration
  - job application
  - employee's Tax Deduction Return.

SOCIAL STUDIES

Students may:

- write an interest inventory test
- investigate the skills/attitudes required in various types of employment
- identify the legal rights and responsibilities of employers and employees
- explore potential employment opportunities in the local community.

OCCUPATIONAL COURSES

Students may:

- identify entry-level skills that are required in a chosen occupational area
  - math skills
  - work skills
  - personal abilities and attitudes
- identify job characteristics within a chosen occupational area:
  - employment trends
  - wages
  - hours of work
  - working conditions
- develop job search strategies within a chosen occupational area
- complete an employee's Tax Deduction Return
- identify compulsory deductions on a pay cheque
- maintain a record of employment and remuneration.

COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer a variety of resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing local community resources, and for involving students in the community by way of meaningful activities linked to finding a job and calculating earnings are provided below.

- Provide opportunities for students to investigate employment skills and job features in an occupational area of personal interest through work experience and job shadowing programs.

- Invite representatives of local businesses and industries to discuss:
  - employment opportunities in the community
  - work skills and employer expectations
  - wage scales/salary ranges and methods of pay.

- Obtain sample pre-employment tests from local businesses and industries. Encourage students to examine these tests, and to note the mathematics skills that are used (e.g., computation, problem solving). Discuss the use of pre-employment tests in screening job applicants. Provide opportunities for students to write one or more pre-employment tests.

- Invite members of the local business community to:
  - discuss the features of an employment interview
  - engage students in a "mock" employment interview.

- Invite representatives of local government agencies to answer questions about various aspects of employment:
  e.g.,
  - Canada Manpower (What are current trends in employment opportunities and the job market?)
  - Labour and Employment Standards (What are the responsibilities of employers with respect to minimum wage and working conditions?)
  - Unemployment Insurance Commission (What is the purpose of an Unemployment Insurance premium?)

FINDING EMPLOYMENT

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Careers Chart (see appendix)

SUPPORT RESOURCE

Math Strategies: Problem Solving (computer software)

OTHER RESOURCES

Decisions, Making Personal Economic Choices
Chapter 6 – Planning for the Job You Want
Chapter 8 – Earning a Pay Cheque

Forms in Your Life: A Student Workbook and Guide to Everyday Forms
Form 2 – Employment Registration
Form 3 – Part-Time Job Application
Form 4 – Full-Time Job Application

Math Wise (video program)
01: Organizing Information

Moving Out
Careers (pp. 44-46)

Street Cents (video program)
Jobs

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Develop career planning strategies and job-seeking skills through consultation with the school guidance counsellor and teachers of English, social studies, CALM and the occupational courses. Cooperative planning at the local level will ensure that activities within this theme complement and extend (but do not duplicate) job search skills that are developed in other subject areas.

2. Encourage students to examine personal interests, values, aptitudes and goals that are relevant to planning a career and/or finding a job. Sample activities that may assist students to identify their interests and goals, and evaluate the appropriateness of various types of employment with respect to personal characteristics are provided in *Decisions, Making Personal Economic Choices*, Chapter 6: Planning for the Job You Want:

   - Activity 1 – Exploring Careers
   - Activity 2 – Personal and Career Values
   - Activity 3 – Job Want Ad Report Form
   - Activity 4 – Career Counselling.

3. Obtain copies of Alberta Education's *Student Workbook* for several different occupational courses in the Integrated Occupational Program. (There are 20 occupational courses and student workbooks from which to choose.) Each workbook has an "Occupational Information" section that describes:

   - entry level skills and jobs
   - physical qualifications
   - preparation for employment
   - employment opportunities and wages.

   Ask students to research the information provided about an occupational area of personal interest in one or more of these workbooks.

4. Encourage students to obtain career information by conducting an interview with a person who works in that occupation.

   Sample questions for conducting a career interview are provided in *Decisions, Making Personal Economic Choices* (p. 59). Job-related factors that may be investigated throughout the interview include:

   - skills, training and experience that are required
   - potential remuneration
   - working conditions
   - worker benefits.

   A chart for recording information that is gathered is provided in Resource 1: Career Information.

5. Ask students to survey potential employment opportunities in the local community by examining the classified section of the newspaper, or by arranging visits to local employment agencies (e.g., a Canada Employment Centre). Using information that is gathered, provide opportunities for students to:

   - select a job for which there are several vacancies, and determine the "range" of salaries being offered for this position (see Resource 2: Salary Ranges)
   - compare/contrast potential wages/salaries offered in several occupational areas by computing equivalent hourly, daily, weekly and/or yearly rates of pay (see Resource 3: Comparing Incomes).

6. Through consultation with the English teacher, arrange to have students write letters of application and/or complete application forms for employment opportunities that have been identified.

Obtain copies of the job application forms used by local businesses, or use the specimen forms provided in *Forms in Your Life*:

- Form 2: Employment Registration
- Form 3: Part-Time Job Application
- Form 4: Full-Time Job Application.

Discuss the meaning of common words, phrases and abbreviations that are used on forms that must be completed when obtaining a job (see *Forms in Your Life*, pp. 4-6).

7. Examine specimen pre-employment tests that are used by local employers. Discuss the mathematics skills that are emphasized in these tests (e.g., estimation, mental arithmetic, problem solving), and how certain skills may be used in work-related situations. Provide students with the experience of writing a pre-employment test.

8. Discuss the purpose of a social insurance number (see *Forms in Your Life*, pp. 9-12). Structure discussion using focus questions such as:

   - What is a S.I.N.?
   - Does everyone need to have a S.I.N.?
   - When do you use a S.I.N.?

   Assist students who have not yet obtained a social insurance number to make application for one.

9. Develop and reinforce appropriate attitudes and effective strategies for solving problems. Appropriate resources may include:

   - CCM: Chapter 4 – Problem-Solving Skills and Strategies
   - *Math Strategies: Problem Solving* (computer software).

   Display a "daily puzzle" or "problem of the week" for students to solve. Encourage students to share favourite puzzles or problems with other members of the class on a regular basis.

CLARIFICATION/EXAMPLE

CALCULATOR GAME OF '21' (2 players)

On each turn a player can push only one of the three keys $\boxed{1}$, $\boxed{2}$ or $\boxed{3}$ along with the $\boxed{+}$ key.

The first player to reach 21 is the winner. Ask students to determine the winning strategy.

CALCULATOR GAME OF '50' (2 players)

This game is similar to the game of '21'. Players may use the $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$ and $\boxed{6}$ keys. The first player to reach 50 is the winner.

Refer to the "Problem Solving" section of this manual for additional ideas on problem solving.

COMPUTING INCOME

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 7 – Income

SUPPORT RESOURCE

MYN: Unit 6 – Earning Money

OTHER RESOURCES

Banking, Budgeting and Employment
Part II – Employment

Decisions, Making Personal Economic Choices
Chapter 8 – Earning a Pay Cheque

Forms in Your Life: A Student Workbook and Guide to Everyday Forms
Form 5 – Employee's Tax Deduction Return
Form 6 – Application for a Union Membership

Mathways (video program)

SRA Computer Drill and Instruction, Mathematics, Level D

Useful Arithmetic, Volume 1
Section III – Work Week

Useful Arithmetic, Volume 2
Section III – Earning Money

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Ask students to find employment opportunities in the classified section of the newspaper that offer remuneration that is based on:

   - an hourly rate
   - a weekly, bi-weekly, monthly and/or annual rate
   - commissions and/or contract work.

   Discuss major advantages/disadvantages of each method of pay, and its implications for budgeting.

41

2. Invite representatives of local government agencies (e.g., Labour and Employment Standards) to discuss regulations and answer questions related to:

   - minimum wage
   - overtime wage
   - worker benefits that an employer must provide.

3. Encourage students to keep a time card of "class working hours" throughout the duration of this theme. Ask students to use the information provided on their time cards to:

   - calculate regular hourly wages earned, if paid the minimum wage for time spent in class
   - calculate overtime wages earned, if paid time-and-a-half for time spent on homework
   - determine their "gross" earnings at the end of each week.

   A sample time card is provided in Resource 4: Hours Worked and Gross Pay.

4. Develop and reinforce basic computational procedures that are useful in performing wage calculations throughout the theme. Learning activities that review basic number concepts and computational skills are provided in:

   - *Consumer and Career Mathematics*
     - Chapter 1 (Whole Numbers, Decimals and Fractions)
     - Chapter 2 (Equations, Proportions and Percent)
     - Skills File (pp. 447-458)
   - *SRA Computer Drill and Instruction, Mathematics, Level D* (computer software).

5. Assist students to distinguish between "gross" earnings and "net" earnings by identifying:

   - compulsory deductions that are withheld from earnings (e.g., income tax payments, Canada Pension Plan contributions, Unemployment Insurance premiums)
   - voluntary deductions that employees may wish to have withheld from their earnings (e.g., retirement pension contributions, union dues, medical insurance premiums, disability insurance).

   Provide opportunities for students to examine actual or specimen "statements of earnings" (i.e., pay stubs). Identify compulsory and voluntary deductions listed on each statement, and encourage students to note the difference between gross earnings and net or "take-home"pay. (e.g., What percent of the gross earnings do the compulsory and/or voluntary deductions represent?)

   Sample pay stubs are provided in Resource 5: Statement of Earnings, and Resource 6: Deductions and Net Pay.

6. Invite representatives from local government agencies and trade unions to discuss compulsory and voluntary deductions from earnings. Prior to the visit, encourage students to formulate a variety of questions related to wages and deductions:

   e.g.,
   - What services are provided by the Canada Pension Plan, the Unemployment Insurance Commission, the Worker's Compensation Board, or a local trade union?
   - How might I benefit from the contributions I make to these or other agencies?
   - What factors determine the amount of the premiums, contributions and/or dues that are deducted from my earnings?
   - How can I estimate the amount of each contribution that will be withheld from my earnings?

   A discussion of Form 5: Employee's Tax Deduction Return and Form 6: Application for a Union Membership provided in *Forms in Your Life* may focus attention on other questions that students may wish to ask.

42

7. Using the results of activity 3 above (or another wage calculation), ask students to:

   - determine the amount of compulsory deductions (e.g., income tax, Canada Pension Plan contribution, Unemployment Insurance premium) that would be withheld from their weekly earnings
   - identify voluntary deductions that might further decrease their earnings
   - calculate their weekly "net" pay.

8. Within some income ranges, compulsory deductions may amount to approximately 25% of the gross pay. Using this rule of thumb, ask students to estimate deductions and net pay for a variety of incomes:

e.g.,
- $307 per week
- $708 per month
- $200 bi-weekly
- $1936 per month.

CLARIFICATION/EXAMPLE

A Strategy for Estimating Net Income

Gross pay: round $307 to $300

Deductions: 25% of $300 = $\frac{\$300}{4}$ = $75

Net pay: $300 - $75 = $225.

Encourage students to explain and share personal strategies they may find useful in estimating net pay in these situations.

43

RESOURCE 1: CAREER INFORMATION

Gather information about a career that interests you by interviewing someone who works in that area. Use the chart below to record the information you gather.

JOB TITLE:

1. Description of work duties:

2. Starting and potential income:

3. Job training and educational requirements:

4. Physical and personal requirements:

5. Working conditions:

6. Employment opportunities:

7. Major advantages/disadvantages of this kind of work:

8. Other interesting factors:

RESOURCE 2: SALARY RANGES

A. Identify a job that you are interested in, and for which you feel you may have the necessary qualifications. Find several vacancies for this position through information gathered in the newspaper or obtained through employment agencies. Determine the hourly, daily, weekly and monthly rates of pay for each vacancy found, and record this information in the table provided. Then determine the salary range for this job.

Name of Job: ______________________________

BUSINESS/COMPANY	RATE OF PAY			
	HOURLY	DAILY	WEEKLY	MONTHLY
SALARY RANGE (subtract lowest pay offer from highest pay offer)				

B. Think of three factors that may affect the salary range noted for this job.

1. ____

2. ____

3. ____

RESOURCE 3: COMPARING INCOMES

Identify different employment opportunities that pay on an hourly rate, and by the day, week, month and year. Compare the remuneration offered for each of these jobs by computing equivalent hourly, daily, weekly, monthly and yearly rates of pay.

TYPE OF JOB	RATE OF PAY				
	HOURLY	DAILY	WEEKLY	MONTHLY	YEARLY
Hourly:	▨				
Daily:		▨			
Weekly:			▨		
Monthly:				▨	
Yearly:					▨

QUESTIONS:

1. Which job pays the highest salary?

____

2. Which job pays the lowest salary?

____

3. Give some reasons why these jobs have different salary scales.

____

____

____

____

4. Would you prefer your salary to be based on an hourly, daily, weekly, monthly or yearly rate? Why?

____

____

RESOURCE 4: HOURS WORKED AND GROSS PAY

TIME CARD

EMPLOYEE: *Jane Doe*

Social Insurance Number: *623 189 458X*

Week Ending: *October 27*

	MORNING		AFTERNOON		EVENING		HOURS
	IN	OUT	IN	OUT	IN	OUT	
M	9:00	12:00	13:00	17:30			
T	9:00	11:30	12:00	17:00			
W	8:00	11:00	12:00	16:30			
Th			12:00	17:00	18:00	20:30	
F			12:30	16:30	17:30	21:00	
S							

	HOURS	RATE	EARNINGS	
REGULAR				
OVERTIME				
TOTALS				

RESOURCE 5: STATEMENT OF EARNINGS

Example One:

<table>
<tr><td>Employee</td><td>Net Claim Code</td><td>Gross Pay</td><td>U.I.C.</td><td>C.P.P.</td><td>Income Tax</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="6">Deductions</td></tr>
<tr><td>Medical Insurance</td><td>Union Dues</td><td>Dental Plan</td><td>Other</td><td>Total Deductions</td><td>Net Pay</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Example Two:

<table>
<tr><td colspan="7">STATEMENT OF EARNINGS</td></tr>
<tr><td>Basic Pay</td><td colspan="2">Overtime Pay</td><td colspan="3">Additional Pay</td><td>Gross Pay</td></tr>
<tr><td>Voluntary Deductions</td><td colspan="5"></td><td>Deductions</td></tr>
<tr><td>Compulsory Deductions</td><td>Canada Pension</td><td colspan="3">Unemployment Insurance</td><td>Income Tax</td><td>Deductions</td></tr>
<tr><td>Date of Issue</td><td colspan="5">Employee</td><td>Net Pay</td></tr>
<tr><td>Pay Period</td><td>Employee #</td><td colspan="2">Dept. #</td><td colspan="3">Employer</td></tr>
</table>

RESOURCE 6: DEDUCTIONS AND NET PAY

Serena quit school last year and found a job at $4.50 per hour for a 35-hour work week. She calculated that every two weeks she would receive $315.00. After paying for room and board, she felt that she would have enough money left over for other expenses.

When Serena received her first pay cheque, she was surprised to see that it was for $248.54 rather than the $315.00 which she had expected. A copy of the Statement of Earnings that was attached to her cheque is provided below.

Examine Serena's Statement of Earnings for her first month of work. Then answer the questions that follow.

STATEMENT OF EARNINGS				
Basic Pay 315.00	Overtime Pay		Additional Pay	Gross Pay 315.00
Voluntary Deductions	(1) 8.06 (2) 1.40 (3) 1.86 (4) (5) (6) 0.94 (7) (8) (9) (10)			Deductions 12.26
Compulsory Deductions	Canada Pension 4.75	Unemployment Insurance 3.02	Income Tax 46.43	Deductions 54.20
Date of Issue **90-06-30**	Employee Serena Smith			Net Pay 248.54
Pay Period 06	Employee # 0010101010	Dept. #	Employer ABC Trucking	

1. Serena had forgotten that certain deductions are made from all wages. Provide the amount deducted for:

   (a) Canada Pension Plan ____________
   (b) Unemployment Insurance Premiums ____________
   (c) Income Tax ____________

2. Other voluntary deductions were subtracted from her gross pay. Suggest some reasons why these deductions might have been made.

   (a) ____________________ (c) ____________________
   (b) ____________________ (d) ____________________

THEME B
BUDGETING AND BANKING

SUB-THEMES

- USING BANK ACCOUNTS
- PREPARING BUDGETS

RATIONALE

Many high school students are looking ahead to the day when they begin full-time employment. In preparing for future employment, students need to become aware of strategies they can use to manage their earnings. This theme will enable students to:

- develop an understanding of the services offered by local banks and other financial institutions, as well as the ability to use appropriate banking procedures
- recognize the value of a personal budget, and develop strategies for maintaining records of money spent and making plans for future saving and spending.

Personal finance skills developed within this theme are intended to establish a foundation for further investigations in Mathematics 26, where students will develop an understanding of the cost of credit and strategies for saving and investing money.

Cooperative planning among teachers will ensure that budgeting and banking activities within this theme complement related topics that are studied by students in other subject areas. The local community offers a variety of resources that will assist students in developing strategies for managing money earned.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed in this theme can be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Assessment/Evaluation.

THEMATIC OBJECTIVES

USING BANK ACCOUNTS

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- identify and compare the purposes of different types of bank accounts
  - personal chequing
  - regular (non-chequing) savings
  - daily interest savings
  - daily interest chequing/savings
- develop an understanding of banking procedures by
  - completing an application for a bank account
  - completing deposit and withdrawal slips
  - completing cheques
  - explaining basic procedures to follow in using automated-teller machines
- maintain a personal record of deposits, withdrawals, and cheques that have been written
- interpret entries in a bank statement and savings account passbook, and reconcile balance with personal records
- compare the costs of various banking services, and determine the advantages/disadvantages of maintaining particular types of accounts
- identify and compare current interest rates established among local financial institutions
- calculate simple interest on money saved and money borrowed for periods of time involving days, months and years
- determine compound interest on money saved and money borrowed through the use of compound interest tables and/or a computer.

PREPARING BUDGETS

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- develop an understanding that a personal budget is useful in accommodating both long-term and short-term needs
- maintain a balance sheet based on personal income and expenditures
- distinguish between regular fixed and discretionary expenditures, and compare the portion of income that is spent on each
- plan for future saving and spending by designing and balancing a personal budget.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

Students will be expected to:

- evaluate banking services offered in the local community by considering the advantages/disadvantages of maintaining particular types of accounts
- compare the effects of simple and compound interest on money saved and money borrowed over varying periods of time
- design a balanced budget that will accommodate personal goals and needs.

USE OF TECHNOLOGY

Students will be expected to:

- use a calculator when reconciling bank balances/statements, designing a balanced budget, and calculating simple interest
- work independently with prepared computer software as required to develop/maintain an understanding of mathematical concepts and skills
- develop an understanding of the use of computer technology in determining simple and compound interest, and in maintaining records of personal banking transactions
- use prepared computer software to determine interest on money saved and money borrowed, and/or to design a balanced budget.

COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform calculations with a calculator on a regular basis
- round as appropriate during calculations
- use mental arithmetic where appropriate to expedite solutions to quantitative problems
- estimate portion of personal income that is used for regular fixed and discretionary expenditures
- use estimation to check the reasonableness of calculations and solutions to problems.

SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

Number Systems and Operations	**Ratio, Proportion and Percent**	**Algebra**	Geometry and Measurement	**Data Interpretation and Display**

NUMBER SYSTEMS AND OPERATIONS

Students will be expected to:

- read, write and order whole numbers, decimals and fractions in applications
- identify place value from 0.01 to 1 000 000
- round whole numbers and decimals as required
- perform basic operations with whole numbers and decimals
- perform a sequence of operations in correct order
- use fractions to represent parts of a whole
- convert fractions into decimals
- use integer concepts/skills as required in balancing a budget and determining a bank account balance.

RATIO, PROPORTION AND PERCENT

Students will be expected to:

- represent whole number percents as ratios and decimals
- represent ratios and decimals as percents
- estimate and calculate a percent of a number
- estimate and calculate what percent one number is of another.

ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions/equations that describe practical situations
- apply equation-solving strategies to practical situations
- interpret formulas as word statements (e.g., $I = prt$)
- use substitution to find the missing element in a formula.

DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- interpret and/or construct tables, charts and graphs that display information about income and expenses
- use interest and amortization tables in practical situations
- interpret and/or calculate appropriate measures of central tendency in practical situations.

INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with budgeting and banking activities that are undertaken by students in other subject areas. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

ENGLISH

Students may:

- identify both long-term and short-term needs that must be accommodated in a personal budget
- develop an understanding of words and phrases frequently used when performing bank transactions (e.g., debit, credit, post-dated cheque, service charge)
- interpret and/or complete a variety of bank forms
  - application for a bank account
  - deposit and withdrawal forms
  - cheques
  - bank statements.

SOCIAL STUDIES

Students may:

- identify factors that need to be considered in managing personal finances
- identify services offered by financial institutions within the local community
- read and interpret articles about budgeting and banking in the financial sections of newspapers and magazines
- consider the advantages/disadvantages of using various forms of credit.

OCCUPATIONAL COURSES

Students may:

- apply budgeting skills when selecting and costing materials used in construction and/or repair projects
- design a financial plan prior to pursuing an entrepreneurial activity.

COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer a variety of resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing local community resources, and for involving students in the community by way of meaningful activities linked to budgeting and banking are provided below.

- Identify various financial institutions in the local area (e.g., banks, credit unions, loans companies). Discuss the different types of services offered by these institutions.

- Invite guest speakers from local banks and credit unions to discuss:
  - the features and purposes of major types of accounts
  - current interest rates paid on money deposited and on money loaned
  - appropriate steps to follow in completing various banking forms and transactions.

- Invite an accountant or financial planner to speak to the class about the importance of financial planning in accommodating long-term and short-term needs. This person may discuss strategies for managing earnings and preparing a personal budget.

- Visit a local bank and observe the use of computer technology. Through interviews with bank personnel, investigate the nature of the tasks performed by computers. What tasks do computers accomplish for:
  - the bank employee?
  - the customer?

USING BANK ACCOUNTS

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 10 - Personal Banking

SUPPORT RESOURCE

MYN: Unit 7 - Savings and Interest

OTHER RESOURCES

A Student Guide to Banking: Students' Portfolio

Banking, Budgeting and Employment
Part I - Banking, Budgeting

Decisions, Making Personal Economic Choices
Chapter 12 - Saving, Spending or Borrowing Money
Chapter 16 - Shopping for Financial Services

Forms in Your Life: A Student Workbook and Guide to Everyday Forms
Form 8 – Application for a Personal Chequing Account
Form 9 – Making Deposits
Form 10 – Writing Cheques
Form 11 – Monthly Statements
Form 12 – Application for a Savings Account
Form 13 – Savings Account Deposit Slip
Form 14 – Savings Account Withdrawal Slip

Math Wise (video program)

Spreadsheets for Students (computer software)

SRA Computer Drill and Instruction, Mathematics, Level D

Useful Arithmetic, Volume 2
Section IV - Banking Services

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Identify and compare the purposes and features of major types of bank accounts. Assist students to develop a sufficient knowledge base by:

   - inviting guest speakers from local banking institutions to explain the features of various accounts available to the consumer
   - reading and summarizing the information provided in Resource 1: Using Banking Services.

   Simulate a variety of relevant situations that require students to use critical thinking skills in selecting bank accounts and banking institutions that are appropriate to personal circumstance. Discuss the advantages/disadvantages of various banking alternatives in each situation.

   Ask students with bank accounts to explain the features of their accounts to other members of the class. Encourage other students to open bank accounts by discussing the services and benefits that a bank account may provide.

   Identify "other" services that may be offered by banking institutions to account holders (e.g., traveller's cheques, certified cheques, money orders, safety deposit boxes, payment of utilities).

2. Provide opportunities for students to develop an understanding of banking procedures by simulating situations that require students to complete appropriate bank forms (e.g., account application forms, deposit and withdrawal slips, cheques, cheque registers). Obtain specimen forms for this purpose from local banks, or use those provided in *A Student Guide to Banking: Students' Portfolio.*

   The following bank forms are also provided in *Forms in Your Life*:

   - Form 8 – Application for a Personal Chequing Account
   - Form 9 – Making Deposits
   - Form 10 – Writing Cheques
   - Form 11 – Monthly Statements
   - Form 12 – Application for a Savings Account
   - Form 13 – Savings Account Deposit Slip
   - Form 14 – Savings Account Withdrawal Slip.

3. Assist students to complete accurately the specimen cheques by reviewing:

   - place value concepts
   - spelling skills that are used in cheque-writing.

   Students may experience difficulty in writing the amount of a cheque in word form. A classroom display of words frequently misspelled on cheques will be useful.

   Discuss procedures to follow in:

   - stopping payment on a cheque that has been written
   - voiding a cheque that has been written.

   Additional information on cheque-writing is provided in Resource 2: Using Cheques.

4. Simulate situations that require students to use a cheque register to keep records of deposits, withdrawals and cheques written. Discuss the importance of maintaining an account balance, and the usefulness of a cheque register in:

   - managing earnings and avoiding overdrafts
   - reconciling bank statements.

Investigate the use of integers to represent an account balance when banking transactions create an overdraft. Discuss the penalties imposed by local banking institutions for overdrafts on an account.

A sample activity that requires students to use a cheque register is provided in Resource 3: Spending Spree.

5. Diagnose individual student strengths and weaknesses with numeration concepts and computational skills throughout the theme. Identify difficulties as they are encountered within banking activities, and provide remediation as required for individual students.

   Learning activities that review and reinforce numeration concepts and computational skills are provided in:

   - CCM: Chapter 1 (Whole Numbers, Decimals and Fractions)
     Chapter 2 (Equations, Proportions and Percent)
     Skills File (pp. 447-458)
   - MYN: Skills Bank
       Unit A - Basic Skills You Need
       Unit C - Computing Skills You Need
   - *SRA Computer Drill and Instruction, Mathematics, Level D* (computer software).

6. Discuss the services provided by automatic teller machines (see Resource 4: Automatic Teller Machines). Identify and describe:

   - basic steps to follow in using an automatic teller machine
   - advantages and disadvantages of using an automatic teller machine.

7. Provide students with a copy of a monthly bank statement and/or several pages from a savings account passbook. Ask students to identify:

   - beginning balance
   - the amounts of cheques written
   - deposits during the month
   - service charges or interest accumulated
   - ending balance.

   Identify the meaning of words, phrases and abbreviations commonly used on bank statements (e.g., debit memo, service charge, entry correction).

   Discuss strategies for reconciling a bank statement with personal records maintained in a cheque register.

8. Develop an understanding of the concept of "interest" on money saved or money borrowed. (A useful analogy might be to compare interest to rent paid for the use of another person's money.) Through class discussion, identify variables that affect interest (i.e., principal, rate and time) and a strategy for determining interest (i.e., I = PRT).

Avoid tedious paper-and-pencil computation of simple interest and keep number relationships simple. When calculating interest, ask questions that focus attention on patterns and relationships among the variables that determine interest. Encourage and provide instruction in the use of calculators, interest tables and/or computer programs when solving routine interest problems.

Additional information that may be used to develop an understanding of the concept of interest is provided in Resource 5: Understanding Interest.

9. Assist students to develop strategies for determining interest with a calculator by:

- reviewing numeration concepts and skills that are used in interest calculations
  e.g.,
  - converting a percent to decimal form
    (18% means 18/100 or 0.18)
  - using a fraction to represent part of a whole
    (4 months = 4/12 of a year)
  - converting a fraction to decimal form
    (4/12 = 4 ÷ 12 = 0.33)
- providing instruction in the use of the percent key. The function of the percent key is not the same on all calculators (see Use of Technology, "The Calculator")
- modelling mental arithmetic and estimation strategies that can be used to check the reasonableness of interest calculations.

CLARIFICATION/EXAMPLE

A STRATEGY FOR ESTIMATING INTEREST

Problem: What is the annual interest on $2450 at 8%?

Estimate: An interest rate of 8% represents $8 of interest on every $100 invested or borrowed.

There are about 25 "hundreds" in 2450.

Therefore, the annual interest is approximately $8 × 25 or $200.

10. Ask students to prepare a table that provides the simple interest and account balance at the end of each month for a savings account that pays monthly interest. This table might be made using the computer (see Use of Technology, "The Computer").

CLARIFICATION EXAMPLE

MONTHLY INTEREST ON A SAVINGS ACCOUNT			
Principal	Interest Rate	Simple	Account Balance
$100.00	5%	$5.00	$105.00
$105.00	5%	$5.25	$110.25
$110.25	5%	$5.51	$115.76
.	.	.	.

11. Ask students to identify potential "bugs" in the following BASIC program designed to calculate simple interest.

```
10 REM     CALCULATE SIMPLE INTEREST
20 PRINT   "WHAT IS THE INTEREST RATE?"
30 INPUT   R
40 PRINT   "WHAT IS THE PRINCIPAL?"
50 INPUT   P
60 PRINT   "THE INTEREST IS:"
70 PRINT   R*P
80 END
```

With assistance, students might adapt this program to calculate simple interest for periods of time other than one year (see Use of Technology, "The Computer").

12. Discuss the meaning of "compound interest", and the advantages/disadvantages it may offer the consumer in situations where money is invested or loaned. Demonstrate, through the use of a table, how compound interest is really a series of simple interest calculations.

CLARIFICATION/EXAMPLE

Period	Principal	Interest Rate	Time	Interest (Prt)	Amount A=P+I
1					
2					
3					
4					
5					
6					

Lengthy computations that often arise from compound interest should be avoided. Encourage students to use compound interest tables and/or computer programs when solving problems that involve compound interest.

13. Provide opportunities for students to investigate and compare the effects of simple and compound interest on money placed in a savings account through the use of *Spreadsheets for Students* (computer software). This computer program removes the tedious arithmetical processes often associated with interest, and focuses attention on elements of problem solving (e.g., critical thinking, estimation, prediction). Blackline masters accompany the program and offer a variety of student activities.

PREPARING BUDGETS

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 9 - Budgeting

SUPPORT RESOURCE

Money Manager (computer software)

OTHER RESOURCES

Banking, Budgeting and Employment
Part I - Banking, Budgeting

Managing Lifestyles: Survival Math Skills
(computer software)

Spreadsheets for Students (computer software)

Street Cents (video program)

Taking Charge of Your Money

Useful Arithmetic Skills
Section III - Earning Money

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Ask students to keep a record of their personal spending for a period of several weeks. Classify expenditures into general categories (e.g., food, clothing, recreation, entertainment, travel, gifts). Encourage students to become aware of how they spend their money.

   Students may wish to record their expenditures using forms similar to those provided in Resource 6: Expense Diary, and Resource 7: Where Does Your Money Go?

2. Ask students to identify sources of personal income (e.g., allowances, money from part-time and occasional jobs, monetary gifts), and to compare their personal income with the expenditures they recorded in Activity 1 above. Display data on personal income and expenditures using a table similar to the one illustrated, or by using Resource 7: Where Does Your Money Go?

CLARIFICATION/EXAMPLE

Date	Details	Income	Expenditures
Oct. 10	Balance on hand	$60.25	
Oct. 10	Lunch		$ 2.20
Oct. 11	Cassette tape		14.95
Oct. 11	Lunch		2.85
Oct. 12	Shirt		28.00
Oct. 12	Baby-sitting	$24.00	
Oct. 14	Lunch		1.80
Oct. 16	Movie		12.00
	TOTALS	$84.25	$ 61.80

Students may wish to enter the data they collect on personal income and expenditures into the computer program *Spreadsheets for Students* (Dynabudget). This program will enable students to display their data in spreadsheet format, and perform a variety of calculations related to income and expenditures that are entered.

3. Discuss the fact that a significant part of personal expenditure is often used for pleasure (i.e., discretionary expenses) rather than for absolute necessities (regular fixed expenses). In the budgeting process, one should begin by planning for necessary expenditures, and determine pleasure expenditures according to the balance.

   Ask students to compare the portion of their personal income spent on necessary expenditures with that spent on discretionary items (see Resource 7: Where Does Your Money Go?). Students can readily determine the portion of their income used for various necessary and discretionary expenditures by entering their data into the computer program *Spreadsheets for Students* (Dynabudget), and running the program in the percent mode.

   Encourage students to analyze and assess personal data related to necessary and discretionary expenditures by discussing:

   - potential problems in spending patterns and routines
   - alternative strategies for resolving these problems.

   A case study that may assist students to assess their own spending routines is provided in Resource 8: Managing Your Money.

4. Ask each student to design a balanced budget based on anticipated income and expenditures for several weeks or months. Remind students that a balanced budget is one in which income exceeds expenditures. Discuss budgeting strategies that are described in Resource 9: Planning a Budget.

   The booklet *Taking Charge of Your Money* provides a number of practical suggestions for designing a personal budget. Topics within the booklet that are particularly relevant to learning objectives within this theme include:

   - Taking Charge (pp. 3-12)
   - Putting Your Spending Plan on Paper (pp. 13-20)
   - So You Want to Cut Down (pp. 31-37).

The computer software *Spreadsheet for Students* (Dynabudget) may also assist students in designing a personal budget. This program enables students to concentrate on problem-solving aspects of the budgeting process by eliminating the need for tedious calculations.

5. Discuss the benefits of a regular saving plan in providing for:

   - major purchases (e.g., car, stereo)
   - future education
   - emergencies (e.g., illness, unemployment)
   - holidays and travel.

   Activities similar to the one below will illustrate the effectiveness of a savings plan in providing for major purchases. Encourage students to "shop" for major items they would like to purchase in the newspaper, and devise appropriate savings plans that will permit the purchase of items selected.

   CLARIFICATION/EXAMPLE

Items I would like to buy	Cost	Amount I can save per month	Number of months I will have to save
A. ______________	______________	$5.00	______________
B. ______________	______________	$8.00	______________
C. ______________	______________	$10.00	______________

6. Ask students to prepare a budget for a simulated "moving out" or "on your own" experience that they may realize in later years (see Resource 10: On Your Own).

   Provide opportunities for students to collect data they require about living costs and future earnings by referencing local newspapers and conducting interviews in the community. Expenditures within the budget might be classified according to:

   - shelter
   - food
   - clothing
   - transportation
   - utilities
   - discretionary spending
   - savings (for insurance, major purchases, holidays, etc.).

7. Assist students to develop an understanding of the usefulness of a budget in planning for future expenditures by investigating current costs related to housing, transportation, home furnishings and/or recreational travel (see Resource 11: Major Purchases). Gather information using the classified section of the newspaper, and illustrate how a regular savings plan may accommodate some of these major purchases.

RESOURCE 1: USING BANKING SERVICES

Banks serve two main purposes. They keep money for people, and they lend money to people. Banks also provide many other services that deal with money. Examples of these services include:

- keeping money safe from theft
- transferring money from bank to bank
- changing currencies (e.g., Canadian dollars to American dollars).

TYPES OF BANK ACCOUNTS

Most banks offer these accounts:

Regular (non-chequing) Savings Account

- a passbook is provided to customers as a receipt, updated as often as you wish
- only cash withdrawals are allowed (no cheques may be written)
- interest is calculated on the minimum (smallest) balance for the month and is paid every three or six months
- interest rates in most cases are higher than in a Daily Savings Account.

Daily Interest Savings Account

- usually a passbook (a record book for the customer) is provided
- only cash withdrawals are permitted (some banks allow a few cheques)
- interest is calculated on the balance at the end of the day
- interest is paid to your account at the end of every month, every three months or every six months (depending on the bank)
- interest rates are slightly lower than the Regular Savings Account rates. This is still a good account for the people whose savings account balance goes up and down during the month
- some banks require a minimum balance to receive the interest rate quoted.

Daily Interest Chequing/Savings Account

- cheque and cash withdrawal privileges
- cheques are stored at the bank
- transactions are recorded in a personal passbook
- interest is calculated on each day's final balance and is paid at the end of each month
- no service charge for cash withdrawals
- service charge for each cheque written.

Personal Chequing Account

- a "demand" account (no waiting time imposed on any amount you want from your account)
- write cheques (no withdrawal slips used)
- record book with blank cheques provided
- statement with cancelled cheques mailed to customer
- no interest paid
- some banks have a service charge for each cheque written.

Term Deposits and Guaranteed Investment Certificates (GICs)

- savings accounts for a fixed length of time
- usually a minimum amount must be deposited
- terms can vary from one day to several years
- the rate of interest is fixed (does not change) and is often higher than the interest rates paid on regular savings accounts
- sometimes these accounts cannot be cashed in until their final date. In other cases, early cashing in can result in a penalty.

From *Mathematics: Applications and Information for the Adult Consumer*, an Adult Basic Education Project (1986) by the Alberta Vocational Centre/Community Vocational Centres. Reprinted with permission.

RESOURCE 2: USING CHEQUES

A cheque is a written order telling a bank to pay the amount of money specified on the cheque to the person named on the cheque.

A chequing account is a bank account where cash withdrawals are not usually made, but cheques are written.

Service Charges

Banks make their money by charging fees for the use of their various services. There may be a charge for each cheque written and/or a monthly fee for the chequing account. The bank may also charge a fee for printing the customer's personalized cheques.

Post-dated Cheques

Cheques written for some date in the future (any date after today) are called post-dated cheques. Many people write post-dated cheques to make regular payments for expenses such as rent. This is generally not a good idea. If you move from your rented home, the landlord might not return your cheques. This could cause you to be overdrawn.

Stale-dated Cheques

Cheques dated more than six months ago are called stale-dated cheques. They are too old or"stale". Such cheques are not cashable. The person who wrote the cheque must either change the date and initial the change or write a new cheque.

Writing a Cheque

Cheques may have scenic pictures or colourful designs on them, but they all ask for the same information. The date, payee (the person the cheque is for), the amount (in numbers and words) and the account holder's signature are to be filled in.

A cheque should always be written in ink, never in pencil. If you make a mistake on a cheque, it's a good idea to tear up the cheque and write another. You may also correct the mistake by crossing it out once, making the correction and writing your initials beside the correction.

It is important to write each cheque in your cheque register right away. If you don't, it can be very difficult to know exactly how much is in your account.

From *Mathematics: Applications and Information for the Adult Consumer*, an Adult Basic Education Project (1986) by the Alberta Vocational Centre/Community Vocational Centres. Reprinted with permission.

RESOURCE 3: SPENDING SPREE

Using a newspaper and the cheques provided by your teacher, you may go on a $1000.00 spending spree. You may purchase anything you wish, but you must write ten cheques. No cheque may be written for less than $10.00. You must also plan a party for six friends and indicate by clipping grocery ads how much is spent on the party. Only one cheque may be written for food. Keep a running balance of money spent on your cheque register. Clip articles of all purchases.

CHEQUE REGISTER

CHEQUE NUMBER	ARTICLE	COST	BALANCE
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

From *That Figures, A Math Resource Package for Intermediate Grades* by Marilyn Hintz and Jerry Ziegler. Copyright 1981 by Kitchener-Waterloo Record Ltd. Reprinted by permission.

67

RESOURCE 4: AUTOMATIC TELLER MACHINES

Automatic teller machines can perform many useful services. If you choose to use an ATM, you may be able to make deposits, withdraw cash, pay bills, transfer funds among accounts, and inquire about your bank balance.

The machines are not difficult to use and careful instructions are given on the machine. First, you insert a plastic access card which has been magnetically encoded with your account number(s). Next, you punch in your personal identification number (PIN). This lets the computer know who you are and that you have a right to access that particular account. Then, you continue with your banking. That is all there is to it. When you finish a transaction, you get a receipt. The receipt notes the type of transaction, the amount involved, the current balance in the account, and the date and time the transaction was made. Don't throw the receipt away.

Important Considerations

- It is very important that you keep accurate records. Be sure to keep the slip you get from the ATM and reconcile it with your monthly statement.
- If there is an error in your account, you may be responsible for bringing it to the attention of the financial institution. Some financial institutions impose a time limit for such a correction.
- Don't write your PIN on your access card. If your card is lost or stolen, your bank account could easily be accessed by a thief. You may be liable for the complete loss because your PIN was written on your access card.
- If you decide to get a card, be sure to ask about transaction charges and fees, and get any agreement in writing.
- Make sure you read and understand the agreement you make with your financial institution when you apply for the card.
- Money deposited in the machines may be subject to a hold period until it is verified. The time may vary from institution to institution.
- Be sure to consider that paying your bills through ATM reduces your float time.

From *Electronic Funds Transfer* by Alberta Consumer and Corporate Affairs. Copyright 1985 by Alberta Consumer and Corporate Affairs. Reprinted by permission.

RESOURCE 5: UNDERSTANDING INTEREST

To encourage people to keep their money in bank accounts, banks pay their customers money called "interest". The customers' money is invested and loaned out to other customers, earning income for the bank. Interest paid to customers comes from this income.

Annual Interest
This is called " simple" interest because it is paid on the principal (original) amount only. To find the amount of interest earned, multiply the rate of interest by the principal. This interest is earned on an annual (yearly) basis, but parts of the interest may be paid during the year.
(Formula: Interest = Principal x Rate or $i = pr$)

Semi-annual
Interest is calculated semi-annually (every six months) on a minimum (least amount) balance. This may earn you more money than an annual interest account.

Quarterly Interest
Interest is calculated every three months (quarter of a year). The balance used would be the lowest amount for that three month period. Most banks offering quarterly interest pay the interest at the end of each quarter (of the year). This makes your minimum balance a bit higher for the next quarter's calculation.

Monthly Interest
This is very common on a regular savings account. Normally, interest is calculated on the lowest balance from the first working day of the month to the last working day of that month. It is paid to the customer either at the end of the month or every six months (semi-annually).

Daily Interest
These accounts have been made possible by computers. Interest is calculated on what your balance is at the end of each day, not on what you deposit or withdraw during the day. Normally banks will calculate on the balance in your account at 3:00 p.m., even if their closing time is later than that. If you are the type of person who withdraws as often as you deposit, a daily interest account is what you need.

Compound Interest
When interest is calculated on a principal amount PLUS the interest, this procedure is called compounding the interest. When it is paid to an account, it is called "compound interest". The more often interest is compounded, the more money you'll earn. People who can afford to bank a fixed sum each month and allow the interest to add up year after year will be surprised to see how fast their investment will grow.

CALCULATING SIMPLE INTEREST

Simple interest is the most basic type of interest. Simple interest can be calculated by using this formula.

$$\text{INTEREST} = \text{PRINCIPAL} \times \text{RATE OF INTEREST} \times \text{TIME}$$
$$I = PRT$$

RESOURCE 5: UNDERSTANDING INTEREST (continued)

For example, Mr. Jones bought a car. He borrowed $2000 at the interest rate of 18% per year. How much:

a) was the interest after one year?
b) was the interest after six months?
c) did he owe after six months?

a) $I = PRT$
$= \$2000 \times 18\% \times 1$
$= \$2000 \times \frac{18}{100} \times 1$
$= \$2000 \times 0.18 \times 1$
$= \$360$

b) $I = PRT$
$= \$2000 \times 18\% \times \frac{6 \text{ months}}{12 \text{ months}}$
$= \$2000 \times \frac{18}{100} \times \frac{6}{12}$
$= \$2000 \times 0.18 \times 0.5$
$= \$180$

c) AMOUNT OWING = PRINCIPAL + INTEREST
= $2000 + 180
= $2180

COMPARING INTEREST RATES

It's a good idea to compare interest rates on loans. Getting the best rates always saves money.

For example. Pat wants to borrow $2000. One company will lend it to her for 17%. Another company will lend it to her for 16%. What is the:

a) cost of each loan over one year?
b) difference in the costs of the loans?

a) COST OF LOAN = PRINCIPAL X RATE OF INTEREST
= $2000 X 17%
= $2000 X 0.17
= $340

COST OF LOAN = $2000 X 16%
= $2000 X 0.16
= $320

b) DIFFERENCE IN COST = LARGER COST - SMALLER COST
= $340 - $320
= $20

CALCULATING MONTHLY INTEREST

Loans paid back in installments are called "amortized" loans. The interest part of each installment is calculated on the unpaid balance, not on the original principal. The interest is calculated on a monthly basis.

Therefore, 12% per year = $\left(12 \times \frac{1}{12} \right)$ % per month = 1% per month

For example, Joe has a loan of $500 at a rate of 18% per year. How much is his first monthly interest payment?

$$\text{MONTHLY INTEREST} = \text{PRINCIPAL} \times \text{RATE OF INTEREST} \times \frac{1}{12}$$

$$\begin{aligned} \text{Monthly Interest} &= \$500 \times 18\% \times \frac{1}{12} \\ &= \$500 \times \frac{18}{100} \times \frac{1}{12} \\ &= \$500 \times 0.18 \times 0.08 \\ &= \$720 \end{aligned}$$

QUESTIONS:

1. What would be the interest on a loan of $3650 at a rate of 18% for one month?

2. Joe needed a short-term loan of $8190 for one month. How much would he pay for the loan at a rate of 16%?

3. How much would $1275 borrowed for one month at 17% cost?

From *Mathematics: Applications and Information for the Adult Consumer*, an Adult Basic Education Project (1986) by the Alberta Vocational Centre/Community Vocational Centres. Reprinted with permission.

RESOURCE 6: EXPENSE DIARY

Date	ITEM...WHAT? PURPOSE...WHY? WHERE? DESCRIPTION? FOR WHOM?	AMOUNT	

From the *Expense Diary* by Alberta Consumer and Corporate Affairs. Reprinted by permission.

RESOURCE 7: WHERE DOES YOUR MONEY GO?

How much money do you spend during a week? Record your estimate of weekly spending in the space provided.

Estimate: ______________________

Complete the record of expenses on the next page over a period of one or more weeks before finishing this activity.

- -

1. How close was your estimate of weekly spending to the actual amount of money spent during one week? ____________

2. Classify your expenses in the table below:

EXPENSES	**Week 1**	**Week 2**	**Week 3**	**Week 4**
Food				
Recreation				
Personal Purchases				
Clothing				
Transportation				
Savings				
Other				

3. Which items would you identify as regular fixed expenses, and which would you identify as discretionary expenses?

____

____

RECORD OF EXPENSES

Days of the Week	Week 1		Week 2		Week 3		Week 4	
	Item	Amount	Item	Amount	Item	Amount	Item	Amount
Monday								
Tuesday								
Wednesday								
Thursday								
Friday								
Saturday								
Sunday								

RESOURCE 8: MANAGING YOUR MONEY

A. Although most families do not spend more than 25% of their income on food, many teenagers do. You are the only person who can decide whether or not you are paying too much for food or for any other item. But how are you to decide?

To make decisions of this type, it is helpful to do some planning or budgeting. The first step is to keep a record of expenses and income over a period of time. The following example provides a record of a Grade 10 student's income and expenses for one month.

INCOME		EXPENSES		
Part-time job		Food:	Pizzas, snacks	$25.00
($20 per week)	$80.00		school lunches	15.00
Allowance		Clothes		20.00
($6 per week)	24.00	Recreation and entertainment		30.00
Odd jobs	10.00	Transportation		15.00
TOTAL	$114.00	TOTAL		$105.00

This student had the satisfaction of knowing that expenses for the month were $9.00 less than income. However, suppose the student wanted to buy a camera that cost $110.00. It would be disappointing to discover that at a rate of $9.00 per month it would take over 12 months to save enough money.

This student asked for an increase in the weekly allowance received, but the request was denied. Spare time did not permit the student to take on any additional jobs that might increase income.

Faced with this dilemma, the student decided to reduce expenses.

Suggest some ways that this might be done.

B. Keep a record of your income and expenses for one month and determine which is the greater. After considering this information, formulate your plans for the next month. Describe any changes you may wish to make in your saving and spending habits as a result of information gathered throughout the first month.

RESOURCE 9: PLANNING A BUDGET

For most people, budgeting means detailed recording of all money spent, cutting back on money spent, and doing a lot of paper work. However, if done right, budgeting can be very little work. It is a way to provide money for things you really want, and an excellent way for you to know how your money is being spent. More simply, a budget is a record of the money you get and the money you spend. It is a plan for future spending.

There are five steps that will make budgeting a simple process.

Step One: Set Goals for the Future

As the family planner you decide what your family needs and wants in the next few months, in the next year, and in the more distant future. It is a good idea to write your goals down in order of importance. Goals for the next 12 months will be very definite. For example, you might decide to pay off $1000 you owe on a credit card account. You will be less sure of your long-term goals, but try to record these also.There are factors you must consider in thinking about your future goals. Is your income likely to change in the next few years? Do you plan to stay home and raise a family? Will you quit working and go back to school? The income you will have in the future will affect the future goals you set.

Step Two: Determine Your Income

It is a good idea to use a budget worksheet as a record and plan for your spending. A sample budget sheet is provided within this resource. You should write down each earner's monthly take-home pay. Take-home pay (or net pay) means the amount leftover after income tax, unemployment insurance, pension, etc., are deducted. You should list only regular income, not money you receive occasionally.

Step Three: Determine Your Expenses

Record all expenses including fixed and irregular monthly expenses and irregular annual expenses. All these expenses together make up the monthly cost of living (i.e., what it costs to live each month). Fixed monthly expenses are those that occur every month and are the same amount each month (e.g., rent). Irregular monthly expenses are expenses that are different each month (e.g., utilities, clothing, personal allowance, credit card payments). Expenses that occur once or twice a year are irregular annual expenses (e.g., car insurance or repairs, Christmas holidays). It is important to list all irregular annual expenses even if you have to estimate the amounts of some. Once you have totaled the irregular expenses, you should divide this amount by 12 and put aside money each month for irregular annual expenses.

When recording expenses gather all your cancelled cheques. Search for any receipts and credit card statements you have kept. It is important to write down what you actually spent, not what you think you spent. After you record all expenses, add them together to get a total for your monthly cost of living.

Step Four: Balancing Your Budget

Look at the amount of your income. Now look at the total for your monthly cost of living. How do they compare? If your expenses are more than your income, changes will have to be made. You will have to spend less money in certain areas or increase your income in some way to make them balance. If your income is more than your expenses, you are living within your budget.

The extra money remaining after paying all your expenses could be put into a savings account. This is one way of providing money for some of your future spending plans or goals. An emergency fund could also be set aside. For example, the money could be used for income if you suddenly become ill and are unable to work for a period of time.

Another factor to consider when balancing your budget is deciding how the expenses will be paid. That is, will you use your chequing account, or will you pay by cash? You might already have a savings account that you can use to pay some of the expenses.

Step Five: Follow Your Budget and Make It Work

Work within your budget for at least three months. This will let you know if all expenses have been included and if the amounts are accurate. From time to time, you must review your spending decisions. Question your spending patterns. Is there something else you want or need? Ask yourself two basic questions:

1. Have I saved anything during the year?
2. Am I less in debt?

If you answer yes to one of these questions, your money management plan is working. If you answer "no" to the questions, assess your spending patterns. Only you can decide how best to spend your money.

RESOURCE 9: PLANNING A BUDGET (continued)

FAMILY BUDGET WORKSHEET

1. MONTHLY INCOME SUMMARY

Source	1st Period	2nd Period
Husband		
Wife		
Family Allowance		
Other		
Sub-total		
TOTAL MONTHLY INCOME ______		

2. MONTHLY COST OF LIVING

Item		Amount
Food & Non-Food		
Household Incidentals		
Personal Allowances & Recreation		
Transportation	Car - Gas	
	Bus or Other	
Other Monthly Expenses		
Housing		
Utilities		
Clothing		
Drugs		
Payments		
Savings		
Irregular Annual		
TOTALS		

MONTHLY RECREATION & PERSONAL ALLOWANCE

Monthly Personal Allowance	Husband	
	Wife	
	Child	
	Child	
Recreation/Entertainment		
TOTAL		

IRREGULAR ANNUAL EXPENSES

Insurance	Car	
	Life	
	Property	
	Medical	
Annual Holiday		
Christmas		
Car Maintenance		
Car License		
Membership Fees/Dues		
Dental		
Taxes		
Other		
Yearly Total		

Monthly Requirement $___ ÷ 12 = $___

MONTHLY UTILITIES

Light		
Water and/or Sewage		
Telephone	Local	
	Long-Distance	
Natural Gas		
TOTAL		

NOTE: IN MONTHLY INCOME SUMMARY, 1ST. PERIOD COLUMN IS USED IF YOU RECEIVE A MID-MONTH ADVANCE OR ARE PAID EVERY TWO WEEKS.

From *Mathematics: Applications and Information for the Adult Consumer*, an Adult Basic Education Project (1986) by the Alberta Vocational Centre/Community Vocational Centres. Reprinted with permission.

RESOURCE 10: ON YOUR OWN

Imagine that you have just recently graduated from high school and have been hired by the XYZ Company. After taxes and other deductions, your take-home pay is $900.00 per month. You were living with your parents, but have just recently rented a furnished apartment. You do not own a car, and must take a bus to work. You would like to buy a car as soon as possible. Eventually, you would like to buy some of your own furniture. The XYZ Company pays for your health insurance.

Determine the monthly cost of the items listed in the chart below. Then design a balanced budget that is appropriate to the circumstances mentioned and your take-home pay of $900.00 per month.

MONTHLY BUDGET

Amount per month

Total Income .... $ ________

Rent .... $ ________

Utilities .... $ ________

- telephone
- gas
- electric

Food .... $ ________

Transportation .... $ ________

Clothing .... $ ________

Personal Purchases .... $ ________

Recreation/Entertainment .... $ ________

Gifts .... $ ________

Future Goals/Savings .... $ ________

Emergencies/Savings .... $ ________

TOTAL .... $ ________

RESOURCE 11: MAJOR PURCHASES

Determine the average cost of each of the following items based on the prices stated in three different newspaper advertisements. Use a calculator to perform your calculations. Document the process used in the spaces provided. Be sure to use estimation and mental arithmetic to check the reasonableness of your results.

1. A four-year-old car
2. Rent of a one-bedroom apartment
3. A used motorcycle
4. A two-storey home
5. A holiday of your choice
6. A bedroom suite

From *That Figures, A Math Resource Package for Intermediate Grades* by Marilyn Hintz and Jerry Ziegler. Copyright 1981 by Kitchener-Waterloo Record Ltd. Reprinted by permission.

THEME C
SKILLS FOR THE CONSUMER

SUB-THEMES

- BUYING GOODS AND SERVICES
- OPERATING A VEHICLE

RATIONALE

This theme focuses attention on strategies that will enable students to maximize the value they receive for money spent. Students will develop an understanding of:

- various perspectives that need to be considered in making informed choices about consumer goods and services, as well as skills used in comparative shopping
- the comparative costs of transportation alternatives, with particular emphasis on the cost of operating a vehicle.

Consumer skills developed within this theme are intended to establish a foundation for further investigations in Mathematics 26, where students will investigate costs associated with establishing a residence of their own and buying/owning a vehicle.

Abundant opportunities exist for students to reinforce basic computation and estimation skills. Students will apply these skills to situations involving decimals, fractions, ratio and percent. Measurement skills will be used in making quantitative comparisons among competing consumer products. Problem solving will place emphasis on developing strategies for making informed consumer choices, and will require students to gather, compare and assess information about competing consumer goods and services.

Cooperative planning among teachers will ensure that skills developed within this theme complement consumer topics studied by students in other subject areas. The local community offers a variety of resources that may be used in developing consumer skills through situational and concrete learning experiences.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed in this theme can be found in:

- Problem Solving
- Computational Facility and Estimation
- Situational and Concrete Approaches
- Assessment/Evaluation.

THEMATIC OBJECTIVES

BUYING GOODS AND SERVICES

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- identify various perspectives that need to be considered in making informed choices about consumer goods and services (e.g., need, quality, cost)
- gather, compare and assess information about competing consumer products and services
- calculate the total cost of a product, when given the unit weight cost and/or unit volume cost
- calculate the unit weight cost and/or the unit volume cost of a product, when given the total cost
- calculate the sale price of an article, when given the regular price and the rate of discount (in fraction or percent form)
- calculate the rate of discount on an article, when given the regular price and sale price
- calculate the sales tax on purchases
- compare the cost of similar products by considering unit cost, sale price and/or the use of coupon discounts
- determine the total cost of articles purchased and verify change received in a cash transaction, using computational procedures appropriate to the situation.

OPERATING A VEHICLE

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- calculate the cost of fuel for travelling a given distance in a vehicle
- determine the fuel economy of a vehicle by calculating the litres of fuel consumed per 100 km
- identify different types of vehicle insurance (e.g., liability, collision, comprehensive), and factors that may affect insurance premiums (e.g., type and age of vehicle; age, gender, marital status and driving record of insured; intended use of vehicle; type of insurance)
- determine the financial loss that may result from depreciation of a vehicle over a given period of time
- estimate (or maintain a record of) the monthly costs associated with operating a vehicle, by considering loan payments, insurance premiums, fuel costs, maintenance and repair costs, depreciation, vehicle registration and driver's licence
- compare the monthly cost of operating a vehicle with other transportation alternatives (e.g., walking, using a bicycle or motorcycle, public transit, taxi, renting a vehicle).

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

Students will be expected to:

- gather, compare and assess information about competing products and services in order to make informed consumer decisions
- consider the costs, advantages and disadvantages associated with alternative forms of transportation.

USE OF TECHNOLOGY

Students will be expected to:

- use a calculator to make cost comparisons among competing consumer goods (e.g., unit cost, multiple cost, sale price), and to determine costs associated with operating a vehicle (e.g., fuel consumption, fuel economy, maintenance costs)
- develop an understanding of how computer technology is used in the pricing of consumer products, in maintaining driver records, and in monitoring vehicle insurance claims
- use prepared computer software as required to:
  - maintain/reinforce basic concepts and skills in numeration and measurement
  - develop strategies for comparative shopping.

COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform calculations with a calculator on a regular basis
- round as appropriate during calculations
- use mental arithmetic when appropriate to expedite solutions to quantitative problems (e.g., to determine multiple cost, unit cost, sale price, total cost of a purchase, change due)
- use estimation skills in a variety of consumer situations (e.g., to make price comparisons, anticipate the total cost of a purchase and change due, determine fuel requirements for a given trip).

SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

Number Systems and Operations	Ratio, Proportion and Percent	Algebra	Geometry and Measurement	Data Interpretation and Display

NUMBER SYSTEMS AND OPERATIONS

Students will be expected to:

- read, write and order whole numbers, decimals and fractions in applications
- identify place value from 0.01 to 1 000 000
- round whole numbers and decimals as required
- perform basic operations with whole numbers and decimals
- perform a sequence of operations in correct order
- multiply and divide whole numbers/decimals by multiples of 10
- use fractions to represent parts of a whole
- calculate a fractional part of an amount in applications
- recall decimal equivalents for one-half, thirds, quarters, fifths and tenths
- convert fractions into decimals.

RATIO, PROPORTION AND PERCENT

Students will be expected to:

- compare two quantities in the same unit by writing ratios
- generate equivalent ratios
- calculate rates/unit rates by writing ratios that involve numbers with different units
- write proportions that describe practical problem situations
- calculate the unknown value in a proportion
- represent whole number percents as ratios and/or decimals
- represent ratios and decimals as percents
- estimate and calculate a percent of a number
- estimate and calculate what percent one number is of another.

ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions/equations that describe arithmetical relationships in practical situations
- apply equation-solving strategies to practical situations
- interpret formulas as word statements
- use substitution to find the missing element in a formula.

GEOMETRY AND MEASUREMENT

Students will be expected to:

- identify common SI units of length, capacity and mass
- use appropriate metric units and tools to estimate length, capacity and mass
- convert among SI units of length, capacity and mass as required in applications.

DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- interpret and/or construct tables, charts and graphs that display information about consumer products and services
- interpret and/or calculate appropriate measures of central tendency in practical situations
- conduct a consumer survey or poll, using appropriate methods of gathering, organizing, presenting and analyzing information.

INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with consumer topics that are studied by students in other subject areas. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

ENGLISH

Students may:

- develop an understanding of persuasive language and propaganda devices that are sometimes used in the promotion of consumer products and services
- gather, compare and assess information about competing consumer products and services.

SCIENCE

Students may:

- develop a strategy for selecting personal care and household products on the basis of their active ingredients and ability to accomplish a given task
- consider the advantages/disadvantages of using various consumer products in terms of their effect on personal health and the environment.

SOCIAL STUDIES

Students may:

- develop an understanding of how government legislation controls the labelling of consumer products
- consider the advantages/disadvantages associated with alternative forms of transportation within the local community.

OCCUPATIONAL COURSES

Students may:

- estimate and measure length, capacity and mass, using units and tools appropriate to the situation
- determine multiple cost and unit cost in planning a project and ordering supplies
- calculate percent discount and mark-up in establishing charges for a service rendered or product produced
- develop an understanding of the costs associated with maintaining and/or repairing a vehicle.

COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer a variety of resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing local community resources, and for developing consumer skills through situational and concrete learning experiences in the community are provided below.

- Invite guest speakers from local businesses and industries to discuss:
  - the advertising and promotional practices of their companies
  - applications of computer technology in product pricing and customer transactions.

- Invite representatives of local government agencies (e.g., Consumer and Corporate Affairs) to discuss consumer issues related to the promotion, labelling and/or pricing of products.

- Gather, compare and assess information on competing consumer goods and services available in the local community. Establish an information base by:
  - visiting several local businesses involved in the sale of personal and household products (e.g., supermarkets, drug stores, department stores, hardware stores)
  - examining advertisements for goods and services that are placed in local newspapers and magazines.

- Invite a knowledgeable person from the insurance industry to describe:
  - different types of vehicle insurance
  - factors that may affect insurance premiums.

- Invite a knowledgeable person from the automotive industry to discuss:
  - strategies for monitoring vehicle fuel consumption and fuel economy
  - various costs associated with maintaining a vehicle.

BUYING GOODS AND SERVICES

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 8 – Buying Food and Goods

SUPPORT RESOURCES

MYN: Unit 5 – Spending Money

Money Manager (computer software)

OTHER RESOURCES

Consumer Complaints

"Consumer Talk" Information Sheets:
- *Shop Around*
- *Shopping at Home*
- *Returns*

Decisions, Making Personal Economic Choices
    Chapter 10 – Getting and Using Information
    Chapter 11 – Comparison Shopping
    Chapter 14 – Shopping for Food and Clothing

Managing Lifestyles: Survival Math Skills (computer software)

Mathways (video program)

Math Wise (video program)

Moving Out
    Marketplace (pp. 32-35)

Spreadsheets for Students (computer software)

Street Cents (video program)

Useful Arithmetic, Volume 2
    Section II – Math in the Marketplace

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Identify and discuss various perspectives that need to be considered in comparing and making choices about consumer goods and/or services. Emphasize that while cost is an important consideration, other factors (e.g., need, quality) also need to be examined.

   A variety of perspectives that may inform the decision-making process in consumer situations are examined in:

   - Resource 1: A Guide for the Consumer
   - Resource 2: Probing and Decision Making
   - Resource 3: A Strategy for Making Informed Consumer Choices
   - Resource 4: How to Get More for Your Shopping Dollar.

   Additional strategies for comparing and making choices about consumer goods and/or services are provided in:

   - *Moving Out* (Marketplace, pp. 32-35)
   - the "Consumer Talk" Information Sheets (see *Shop Around*, *Shopping at Home*, and *Returns*)
   - *Decisions, Making Personal Economic Choices* (Chapter 10: Getting and Using Information, Chapter 11: Comparison Shopping, and Chapter 14: Shopping for Food and Clothing).

2. Develop strategies for comparing consumer goods and services by actively involving students in related activities. Provide opportunities for students to:

   - gather information about products or services they frequently use (e.g., food items, cosmetic and personal care items, clothing) by visiting local stores and reading newspaper advertisements
   - choose among particular products or services, after considering a variety of perspectives (e.g., need, features of the product or service, quality, cost).

   Sample activities that require students to become actively involved in comparative shopping are provided in Resource 5: Where Shall I Buy It?

3. Collect and display newspaper advertisements for a number of familiar consumer products and/or services. Within each advertisement, ask students to:

   - underline key words and phrases
   - identify persuasive language that may have been used in attempts to influence the consumer
   - make note of additional information that is needed in order to make an informed choice about the product and/or service.

   A sample activity that illustrates how this strategy can be used in analyzing newspaper advertisements is provided in Resource 6: Buyer Beware.

4. Ask students to select a consumer product they may wish to purchase (e.g., tape player, bicycle). Gather information on several different makes/models of the product by reading advertisements in the newspaper, through the use of consumer catalogues, and/or by visiting local stores. Display information that is gathered in a table similar to the one illustrated below, and encourage students to consider the advantages/disadvantages associated with each alternative.

CLARIFICATION/EXAMPLE

MAKE/MODEL	FEATURES	QUALITY	WARRANTY	COST
Model A:				
Model B:				
Model C:				

Prepare a graph that illustrates price variation among different makes/models of the product.

5. Discuss and model strategies for estimation and mental arithmetic that are appropriate in a variety of practical consumer situations (see Computational Facility and Estimation, "Developing Strategies for Mental Arithmetic and Estimation"). Practical situations that require the use of these skills may include:

- determining the cost of one article, when given the cost of several similar articles

  e.g., "4 for $1.79"
  How much will one cost?

- determining the cost of several similar or different articles, when given individual costs

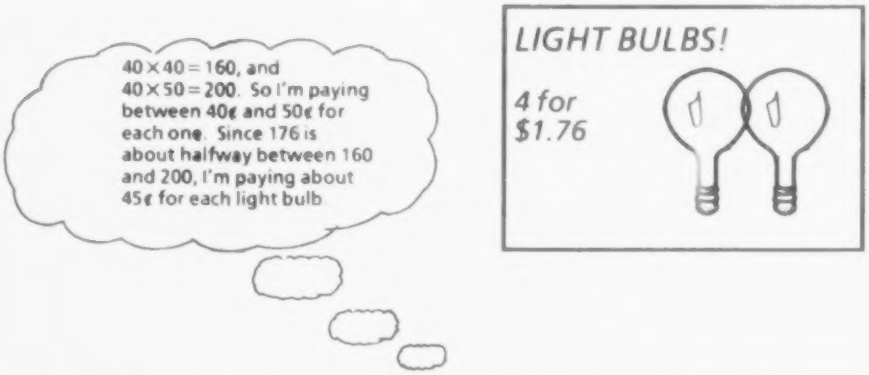
  e.g., "$2.95 each"
  How much will six cost?

- anticipating the amount of change due, when paying for a purchase with cash.

Encourage students to share personal strategies they find useful in these and similar situations with other members of the class.

CLARIFICATION/EXAMPLE

USING ESTIMATION SKILLS


40 × 40 = 160, and 40 × 50 = 200. So I'm paying between 40¢ and 50¢ for each one. Since 176 is about halfway between 160 and 200, I'm paying about 45¢ for each light bulb
LIGHT BULBS!
4 for $1.76

Activities provided in Resource 7: Paying for Purchases, and Resource 8: Sales and Coupons require students to use estimation and mental arithmetic in simulated consumer situations. Additional activities that may be used to develop skills in estimation and mental arithmetic are provided in *Mental Math in Junior High*.

6. Lessons in "unit pricing" should enable students to understand that:

   - unit price indicates how much an item costs per standard unit of measure (e.g., price per gram, per litre or per metre)
   - unit price is used to compare the price of competing products that have different packaging.

   Encourage students to use a calculator when calculating unit price, and develop appropriate strategies for:

   - determining the missing number in a proportion (see Situational and Concrete Approaches, "Ratio, Proportion and Percent")
   - interpreting and rounding numbers displayed on the calculator
   - checking the reasonableness of results obtained on the calculator.

   Activities that require students to determine and compare unit prices for a variety of familiar products are provided in Resource 9: The Better Buy.

7. A review of ratio and proportion skills may assist students to understand unit pricing and solve related problems. Students should understand that:

   - ratios describe pricing rates that involve different units

     e.g., $\frac{\$.95}{250\text{ g}}$ $\frac{\text{dollars}}{\text{grams}}$

   - proportions (equivalent ratios) describe the process of unit pricing

     e.g., $\frac{\$.95}{250\text{ g}} = \frac{?}{100\text{ g}}$

   Learning activities that can be used to develop/reinforce computational skills with ratio and proportion are provided in:

   - *Consumer and Career Mathematics*
     - Chapter 2 (Equations, Proportions and Percent)
     - Skills File (p. 456)
   - *SRA Computer Drill and Instruction, Mathematics, Level D* (computer software)
   - *Math Wise* (videotape)
     - Comparing 01 (Numerical comparisons)
     - Comparing 04 (Proportions).

8. Comparison shopping requires familiarity with common units of length, capacity and mass, and ability to convert between units that are frequently used to measure consumer goods.

   Encourage students to become familiar with common units of measure by displaying containers/items of varying size and shape, and asking students to:

   - estimate their length, capacity or mass
   - measure their length, capacity or mass as a check against estimates.

Ability to convert between units that are used to measure consumer goods requires an understanding of:

- the meaning of metric prefixes (e.g., milli, centi, kilo)
- strategies for multiplying and dividing numbers by multiples of 10.

Activities provided in Resource 10: Unit Pricing By Mass and Capacity require students to become actively involved in estimating and measuring the mass/capacity of familiar products in order to make price comparisons.

Additional learning activities that focus on the development of estimation and measurement skills are provided in:

- *Consumer and Career Mathematics*
  - Chapter 3 (Measurement and Geometry)
- *Math You Need, Book 2*
  - Unit B (Measurement Skills You Need)
- *Math Wise* (videotape)
  - Comparing 03 (Finding a Common Unit).

9. Introduce the concept of "discount" by displaying a variety of newspaper advertisements that describe consumer goods and/or services as being discounted or on sale. Ask students to determine:

- the sale price in situations where the percent discount is given
- the percent discount in situations where the original price and sale price are given.

Model/discuss appropriate strategies for finding discounts and sale prices in these situations.

Example One: An article is on sale at a 25% discount.
The original price was \$8.00.
25% of \$8.00
= 0.25 × \$8.00
= \$2.00

Discount = \$2.00
Sale Price = \$8.00 − \$2.00 = \$6.00

Example Two: An article, originally priced at \$35.00, is on sale for \$24.50.

Discount = \$35.00 − \$24.50 = \$10.50

$$\frac{\$10.50}{\$35.00} = \frac{?}{100}$$

Percent discount = 30%.

Once computational processes are understood, encourage students to use a calculator in performing necessary computations. Provide instruction on using the percent key, and model the sequence of entries that are made in solving these and other problems involving discount and percent (see Use of Technology, "The Calculator").

Activities that require students to determine sale price and percent discount are provided in Resource 11: Sale Prices, and Resource 12: What is the Percent Discount?

10. Some students may experience difficulty with concepts related to percent. Diagnose areas of student strength/weakness, and plan remedial activities that provide concrete and visual experiences with these concepts.

e.g.,
- Students may benefit from pictorial representations of percent using the 10 × 10 grid. Shade the percent of discount in red. Look at the unshaded portion of the grid in order to determine what percent the discounted price is of the original price.
- Develop an understanding of common fraction, decimal and percent equivalents (e.g., one-half, quarters, tenths) through the use of pictures, diagrams and charts displayed in the classroom.
- Develop strategies for solving proportions using the "cross-product" method after students have experience in solving proportions using the common multiple/common factor methods (see Situational and Concrete Approaches, "Ratio, Proportion and Percent").

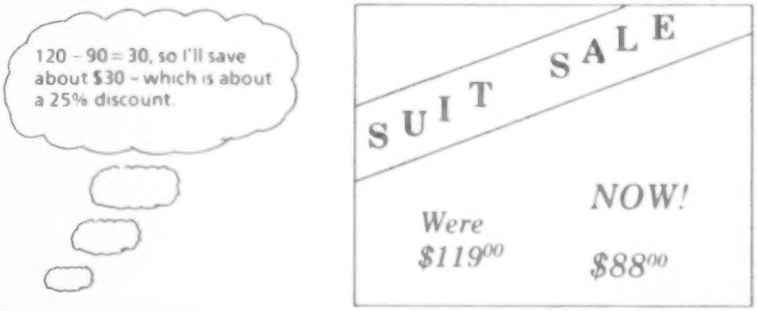
Learning activities that focus on developing an understanding of percent are provided in:

- *Consumer and Career Mathematics*
  - Chapter 2 (Equations, Proportions and Percent)
  - Skills File (pp. 457-458)
- *Math You Need, Book 2*
  - Unit C (Computing Skills You Need)
- *Mathways* (videotape)
  - The Percent
- *Math Wise* (videotape)
  - Comparing 02 (Percent)
- *SRA Computer Drill and Instruction, Mathematics, Level D* (computer software).

11. Model and discuss estimation strategies that are useful in consumer situations involving discounts. Students should recognize that although precise answers are sometimes required, there are many consumer situations in which "ball-park" estimates are appropriate.

CLARIFICATION/EXAMPLE

USING ESTIMATION SKILLS


120 − 90 = 30, so I'll save about $30 − which is about a 25% discount.
SUIT SALE
Were $119⁰⁰
NOW! $88⁰⁰

A variety of strategies for developing estimation skills (e.g., front-end estimation, rounding, compatible numbers, clustering) are provided in another section of this document (see Computational Facility and Estimation, "Developing Strategies for Estimation").

12. The computer program *Spreadsheets for Students* (Dynashop) provides opportunities for students to apply a variety of critical thinking skills to consumer situations. Simulations require students to make informed consumer choices after considering a variety of cost-related factors, including unit cost, total cost, sale discounts and coupon discounts. Tedious arithmetical processes are removed in the program, allowing students to focus attention on problem-solving aspects of each situation.

    The computer program *Managing Lifestyles: Survival Math Skills* (Comparison Shopping) provides opportunities for students to develop useful strategies for grocery shopping. Simulations in this program require students to purchase a week's supply of groceries for a family of four, and focus attention on servings (i.e., the amount of food necessary to serve one person), unit cost and the "best buy".

OPERATING A VEHICLE

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 13 – Automobile Operating Expenses

SUPPORT RESOURCES

MYN: Unit 2 – Automotive Math

Money Manager (computer software)

OTHER RESOURCES

Decisions, Making Personal Economic Choices
Chapter 15 – Shopping for Housing and Transportation

Forms in Your Life: A Student Workbook and Guide to Everyday Forms
Form 16 – Application for a Driver's Licence
Form 18 – Application for Car Insurance

Moving Out

Useful Arithmetic, Volume 1
Section IV – Transportation

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Identify and discuss personal responsibilities associated with operating a vehicle. Discussion may involve the identification of specific responsibilities related to:

   - vehicle maintenance
   - driver knowledge and skills
   - insurance, registration and licensing requirements
   - the cost of operating a vehicle.

   Encourage students to formulate questions related to the responsibilities they identify. Invite a representative from the Motor Vehicles Branch to answer these questions and discuss other factors relevant to operating a vehicle.

Alberta Consumer and Corporate affairs publishes a variety of materials that outline responsibilities associated with operating a vehicle. The information provided in the booklet *Moving Out* (pp. 28-31), and teaching ideas provided in the accompanying *Instructor's Guide* (pp. 47-59) are particularly relevant to this sub-theme.

2. Read the information about vehicle insurance provided in Resource 13: Insuring Your Vehicle. Identify and discuss:

   - the purpose of different types of vehicle insurance
     e.g.,
     - liability
     - collision
     - comprehensive
   - various factors that may affect insurance premiums
     e.g.,
     - type and age of vehicle
     - age, gender, marital status and driving record of insured
     - intended use of vehicle
     - type/amount of insurance.

   The tables provided in Resource 14: Automobile Insurance Rates illustrate how driving record, type of vehicle and amount of insurance coverage may affect the insurance premiums paid by a male driver between the ages of 16 and 25.

   Additional information about vehicle insurance is provided in the booklet *Car Insurance Explained*, available from the Insurance Bureau of Canada at the address provided below.

   The Insurance Bureau of Canada
   1105, 10080 Jasper Avenue
   Edmonton, Alberta
   T5J 1V9
   Phone: 423-2212 or 1-800-232-7275

3. Invite a representative from the Motor Vehicles Branch or an automobile insurance salesperson to discuss:

   - minimum insurance requirements for vehicle operators
   - factors that affect insurance premiums.

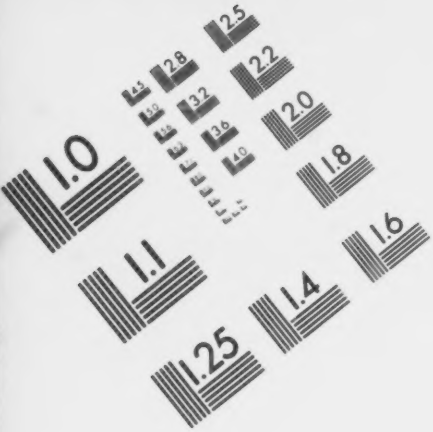
   Ask students to consult two or more local insurance companies in order to determine the cost of similar types of vehicle insurance. Compare insurance costs, and decide if different premiums reflect differences in the coverage or services offered by these companies.

4. Through consultation with the English teacher, arrange to have students examine and possibly complete application forms for a driver's licence, vehicle registration and vehicle insurance.

   Obtain copies of appropriate forms from local agencies, or use the specimen forms provided in *Forms in Your Life*:

   - Form 16 – Application for a Driver's Licence
   - Form 18 – Application for Car Insurance.

   Discuss the meaning of common words, phrases and abbreviations that are used on these forms.

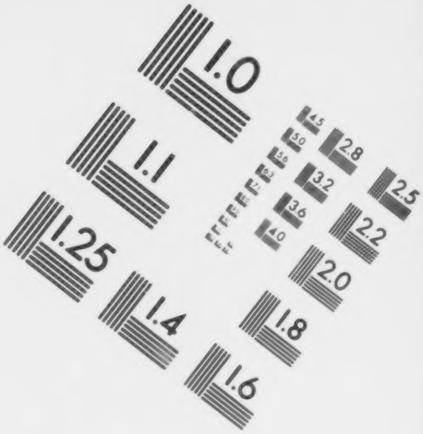


AIIM

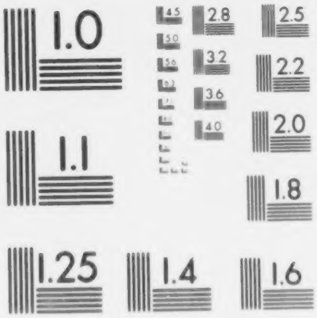
Association for Information and Image Management

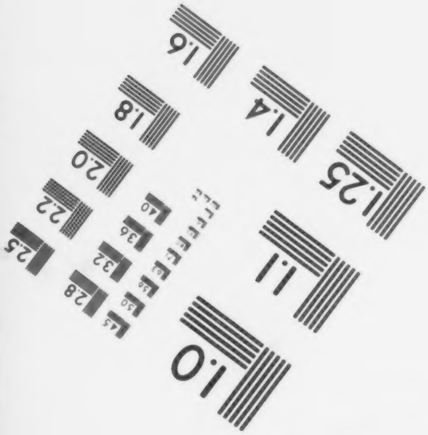
1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910

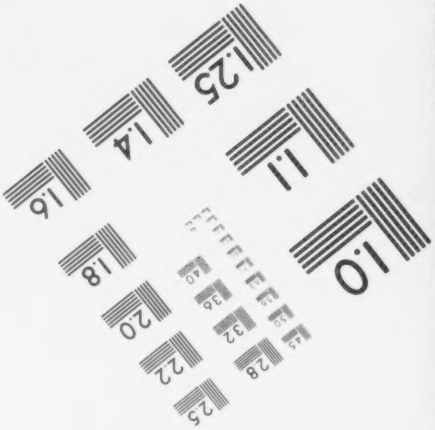
301/587-8202



Centimeter
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 mm
1 2 3 4 5
Inches


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6





MANUFACTURED TO AIIM STANDARDS
BY APPLIED IMAGE, INC.

5. Provide opportunities for students to estimate (or maintain a record of) the weekly/monthly costs associated with operating a vehicle. When estimating operating costs, encourage students to consider:

   - fixed costs
     e.g.,
     - loan payments
     - insurance premiums
     - depreciation
     - vehicle registration and driver licensing
   - variable costs
     e.g.,
     - fuel costs
     - routine maintenance costs
     - other repair costs.

   If possible, invite the Automotive Services teacher or another knowledgeable person in the community to discuss routine maintenance costs incurred when operating a vehicle (e.g., the costs of oil changes, lubrication, engine tune-ups, tire rotation, etc.).

   An analysis of vehicle operating expenses is provided in the Canadian Automobile Association's booklet *Car Costs*. This booklet may be useful in guiding student activities, and can be obtained from the local Alberta Motor Association office, or by contacting:

   Alberta Motor Association
   Consumer Information Service
   P.O. Box 3740, Station "D"
   Edmonton, Alberta
   T5L 4J5
   Phone: 474-8714 or 1-800-222-6578.

6. Discuss the use of ratio in describing fuel consumption (e.g., L/100 km). Investigate the average fuel consumption rates for several different vehicles. Using these rates, estimate/calculate the fuel costs associated with daily travel.

   Invite a local automotives expert to discuss topics related to fuel consumption.

   e.g.,
   - strategies for monitoring/calculating fuel consumption
   - average fuel consumption rates for different types of vehicles
   - factors that may influence fuel consumption (e.g., weight of cargo, driving habits, type of transmission)

   The average fuel consumption rate for many vehicles is provided in the *Fuel Consumption Guide*, available at no charge from Transport Canada, Public Affairs Branch, or through local agencies such as the Alberta Motor Association.

7. Select several travel destinations on an Alberta road map. Through the use of distance charts and average fuel consumption rates, determine:

   - travel distances
   - the approximate number of litres of fuel required to travel these distances
   - fuel costs for each trip (using current fuel prices).

8. Provide opportunities for students to examine the costs associated with other methods of transportation in the local community.

   e.g.,
   - walking
   - bicycle/motorcycle
   - public transit
   - taxi.

   Encourage students to consider the advantages and disadvantages of various transportation alternatives by considering:

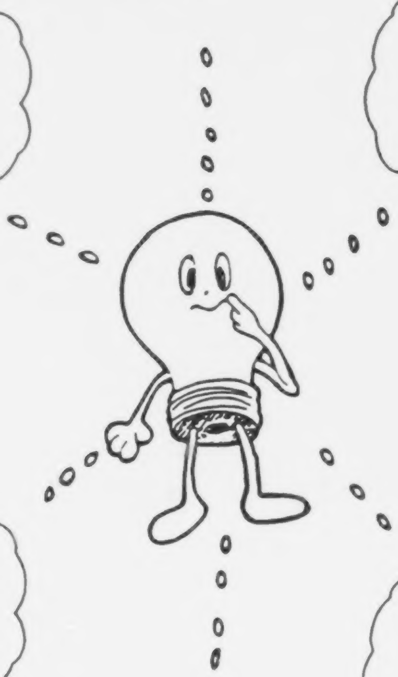
   - cost
   - personal needs
   - comfort and convenience.

RESOURCE 1: A GUIDE FOR THE CONSUMER

Know how to use MONEY
Do I have enough money to buy this? How much change will I get back?

Know your NEEDS
Do I need this now? Is it a priority in my budget?

Know WHERE to find it
Where do I go to find what I'm looking for?



Look for QUALITY
Is this good quality? Is it made well and will it satisfy me?

Judge ADVERTISING
Do I really want and need this, or did I get caught up in the advertising?

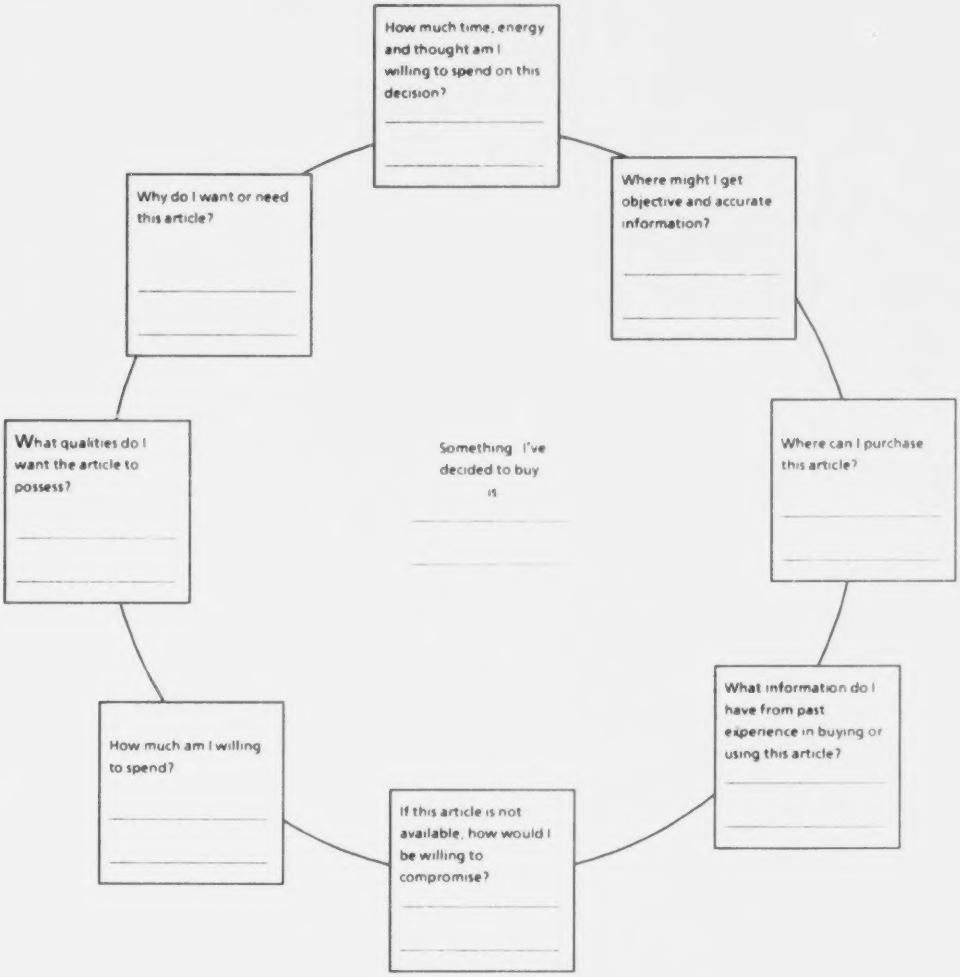
COMPARE carefully
How does this compare with similar ones? Can I get the same quality for a lower price?

RESOURCE 2: PROBING AND DECISION MAKING

Probing (or questioning) is an activity that will assist decision making. By asking questions, we can consciously focus on our alternatives or choices and become increasingly informed prior to making decisions. Probing indicates areas where we may have insufficient information, and also suggests how the information we possess can be used in making a particular decision.

EXERCISE

Think about something you're planning to buy. Write out answers to some questions or probes you might provide yourself before making this decision.


How much time, energy and thought am I willing to spend on this decision?
Where might I get objective and accurate information?
Where can I purchase this article?
What information do I have from past experience in buying or using this article?
If this article is not available, how would I be willing to compromise?
How much am I willing to spend?
What qualities do I want the article to possess?
Why do I want or need this article?
Something I've decided to buy is

RESOURCE 3: A STRATEGY FOR MAKING INFORMED CONSUMER CHOICES

Comparison shopping takes time but there is an obvious payoff. You save money and get a better deal! No matter what you are shopping for, there are five steps to making an informed choice. Ask yourself the following questions:

1. Why am I buying?

   If the item is something you actually need, you can justify the purchase. If you merely want the item, think carefully before buying. It is all too easy to become a chronic impulse spender, buying the things you want as you see them, and then not having enough money left for the things you need.

2. What should I buy?

   Almost anything you decide to buy is available in a wide range of models, brands, sizes and prices. To make the best choice, learn how to read advertising claims and label information. Check your local library for product test reports and relevant books. Collect informative pamphlets from stores, manufacturers, and consumer help agencies. Do not overlook your friends and neighbours. Sometimes the best information comes from people who have had first-hand experience with the product. Finally, do not rely on price alone to tell you which is the best buy. The most expensive item is not necessarily the right choice for you – nor is the cheapest item.

3. When should I buy?

   Prices on almost everything vary throughout the year. Plan your purchase in advance to take advantage of the seasonal and other sales. Be sure that an advertised sale is a genuine sale.

4. Where should I buy?

   Find out where you can get the best servicing (repairs and maintenance), the best warranty and the lowest price for the specific brand and model that you have chosen. If you have not previously dealt with the companies selling the item you plan to buy, ask your friends and neighbours for referrals.

   The Better Business Bureau can tell you whether it has received any complaints against a business. Save footwork by phoning around.

5. How should I pay?

   If you plan far enough in advance, you will likely be able to pay cash for your purchase, or at least make a large down payment. If you decide to use credit, realize that it is just as important to comparison shop for credit as for the item itself. Interest rates and other conditions vary, so shop around to get the best deal.

From *Mathematics: Applications and Information for the Adult Consumer*, an Adult Basic Education Project (1986) by the Alberta Vocational Centre/Community Vocational Centres. Reprinted with permission.

RESOURCE 4: HOW TO GET MORE FOR YOUR SHOPPING DOLLAR

We all have to eat, and we all shop for food. The food bill for an average Canadian family of four is approximately $375.00 per month, or $4,500.00 per year. This is about 18% or more of our disposable income.

In a lifetime, a person may spend in excess of $80,000 on food, more than many of us will spend on a home. So how do we get the best value for our food dollars? It takes time, energy and effort, but it's not impossible. Here are a few things we need to know:

- Grocery stores are in business to make money. The more they can get us to buy, the more money they make. These companies spend billions of dollars each year on advertising. They arrange their stores in such a way as to trick us into "Impulse Buying" – buying things we didn't plan to buy.

- We need to ask ourselves: "Do I buy what I want to have on my cupboard shelves, or am I buying what the stores want to get off their shelves?"

- Nutrition (the nourishing qualities of food) is very important. We must look for the best buy, and also buy those things that have the most food value.

There are many ways you can stretch your budget when shopping. A list of helpful tips for saving money when you are shopping is provided below.

1. Make a list. When food shopping, a list cuts down on "impulse buys". List your items according to store location. This makes shopping less tiring.

2. Check the ads. Weekly food advertisements list specials. Many of these specials are "loss leaders" selling at cost or lower to bring in the crowd.

3. Shop only once a week. The more often you visit a store, the more money you spend on impulse buys.

4. Shop alone if you can. This will enable you to concentrate on the things you really need.

5. Calculate the cost per unit. Large, economy sizes are usually a saving, but not always. Be careful of buying food in quantity. It's no bargain if it spoils before you can use it.

6. Buy cheaper grades and cuts of meat. Cheaper grades and cuts are just as nutritious as the more expensive kind. Look for recipes that call for long, slow cooking which helps tenderize meat.

RESOURCE 4: HOW TO GET MORE FOR YOUR SHOPPING DOLLAR (continued)

7. Buy meat by cost-per-serving, not cost per kilogram. A boneless roast costs more but consider how many servings you can get compared to a bone-in roast.

8. Compare the size and price of eggs. If medium eggs are eight cents a dozen cheaper than large eggs in the same grade, buy medium. They will prove a better buy. The larger size will not reduce the number of eggs you use.

9. Buy fresh foods in season, but not at first. Prices go down as new supplies come in. Out-of-season foods are high priced imports.

10. Buy bulk frozen foods. Frozen fruits and vegetables in big, poly bags cost less. Use what you need and keep the rest frozen.

11. Look for low-cost recipes. You can cut your food bill considerably by using lower-priced foods listed in many recipes found in newspapers, magazines and cookbooks.

12. Check lower-priced brands. Less known brands are often cheaper than nationally advertised brands, and can be equally good.

13. Save time and money. Some convenience foods (e.g., canned or frozen fruit juices, vegetables and some baking mixes) are often cheaper than their fresh counterparts.

14. Check the lower shelves. In most stores, less expensive brands are stocked on lower shelves while more expensive items are placed on easy-to-reach shelves.

15. Reduce the cost of milk by using non-fat dry milk for cooking and to make skim milk. The larger containers of milk save money for families that consume large amounts of milk.

16. Watch the pennies. A few pennies here and there can add up to dollars in your pocket after a month of food shopping.

From *Mathematics: Applications and Information for the Adult Consumer*, an Adult Basic Education Project (1986) by the Alberta Vocational Centre/Community Vocational Centres. Reprinted with permission.

RESOURCE 5: WHERE SHALL I BUY IT?

Identify three local stores that would sell the following items. Find the price of the item at each of the stores. Then answer the questions that follow.

TOOTHPASTE (150 mL):	STORE 1	STORE 2	STORE 3
BRAND A:			
BRAND B:			
BRAND C:			

SPORTS SHOES:	STORE 1	STORE 2	STORE 3
BRAND A:			
BRAND B:			
BRAND C:			

JEANS:	STORE 1	STORE 2	STORE 3
BRAND A:			
BRAND B:			
BRAND C:			

QUESTIONS

1. Where would you shop for each of these items? Give reasons for the purchase decisions you have made. What factors other than price have you taken into consideration?
2. Graph the differences you found in price for at least one of these items.
3. Was there a difference in the general atmosphere of the stores that you visited? Explain.
4. In what way would the following services affect the prices charged by stores?
   a) coffee shops within the store
   b) late night and holiday business hours
   c) number of clerks and sales people
   d) self-serve and self-packing.

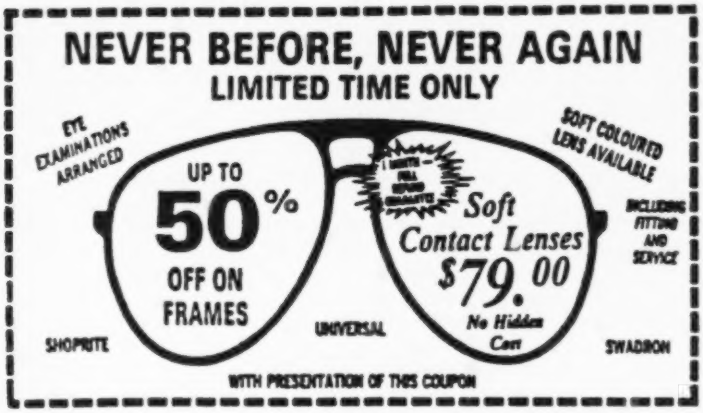
RESOURCE 6: BUYER BEWARE

Read the newspaper advertisements.

Underline all words or phrases in each advertisement that the consumer should be alerted to.

Highlight persuasive language that is used to influence the consumer.

List at least three questions that the buyer should ask before purchasing each item advertised.



NEVER BEFORE, NEVER AGAIN

LIMITED TIME ONLY

EYE EXAMINATIONS ARRANGED

SOFT COLOURED LENS AVAILABLE

UP TO 50% OFF ON FRAMES

Soft Contact Lenses $79.00

No Hidden Cost

INCLUDING FITTING AND SERVICE

SHOPRITE

UNIVERSAL

SWADRON

WITH PRESENTATION OF THIS COUPON

SHAME ON YOU

IF YOU DON'T TAKE ADVANTAGE OF THESE CERTIFIED CARS

81 Horizon 5 Door Custom $5595

Auto, radio, Economical 4 Cyl etc Must be seen SKB991

80 Buick Riviera 2 Door $10,575

2 tone brown, fully powered, one owner low mileage vehicle RBP394

80 Aspen 4 Door $4885

6 cyl automatic, p/s, radio, etc A real beauty PZK112

78 Malibu Classic 4 Door $4499

Air, C.B. radio, p/s, p/b, stereo, etc Seeing is believing! SJN562

77 Volare Wagon $3565

Auto, p/s, Economical 6 cyl roof rack, woodgrain, buckets, etc
Priced right! LKK169

"Financing at BANK RATES available!"

SEEING IS BELIEVING

From *That Figures, A Math Resource Package for Intermediate Grades*, by Merilyn Hintz and Jerry Ziegler. Copyright 1981 by Kitchener-Waterloo Record Ltd. Reprinted by permission.

RESOURCE 7: PAYING FOR PURCHASES

It is often necessary for us to make rapid calculations in our head. For example, in making purchases we must:

- determine the total cost of the items being purchased to ensure that we have enough money to pay for them; and
- verify the change we receive in the purchase transaction.

Practise your skills in mental arithmetic by answering the questions without using paper-and-pencil or a calculator. Once you have determined answers through mental arithmetic, check your accuracy by using the calculator.

1. Mr. Launi wanted to buy a steak for $3.95, a box of laundry detergent for $2.95, a jar of coffee for $4.15, and a jar of mayonnaise for $.95. What was the total cost of these items?

2. Paul bought a baseball glove that was regularly sold for $16.49 at a reduction of $2.98. How much did he pay for the glove?

3. How many 37¢ stamps can you buy for $15.00?

4. Your bill at the supermarket was $24.67. You gave the cashier $30.00. How much change should you receive?

5. Jane bought a ten-speed bicycle. It cost $129.78. She gave the cashier seven 20-dollar bills. How much change should she have received?

6. Your sister bought a hair dryer. It cost $15.95. She gave the cashier four 5-dollar bills. How much change should she have received?

7. You are buying a portable radio that costs $21.98. You have six 5-dollar bills in your wallet. How many bills should you give the cashier? How much change should you receive?

8. You have just purchased two packages of gum at 45¢ each, some toothpaste that costs $2.49, and a 12-pack of pop for $7.98 at Shop-for-Less Supermarket. You are paying with a cheque, but want to receive $15.00 in change. What amount should the cheque be written for?

RESOURCE 8: SALES AND COUPONS

Here are some questions you may have to find answers to at the supermarket. Use mental arithmetic and/or approximation to find exact or nearly exact answers to the questions. Use a calculator to check the answers to questions you are unsure about.

1. The sale price for lettuce is two heads for $1.59. How much will you have to pay if you buy only one head?

2. The sale price for canned peas is two cans for $1.09. How much will you have to pay if you buy only one can?

3. a) Small cans of soda are on sale at eight cans for $3.00. How much will one can cost?

   b) If you went to the store eight different times and bought one can of soda each time at the sale price, how much would the eight cans cost?

4. The sale price for light bulbs is three for $2.30. How much will one light bulb cost?

5. You have a coupon that says **9¢ off any size bottle of Good Home Salad Dressing**. A bottle regularly costs $1.89. How much will it cost with the coupon?

6. You have a coupon that says **Save 13¢ on your next purchase of ABC Spray Cleaner**. A bottle of the cleaner regularly costs $1.09. How much will it cost with the coupon?

7. a) You have a coupon that says **23¢ off on a giant size bag of Big Boy Dog Food.** A giant size bag regularly costs $5.99. How much will it cost with the coupon?

   b) You usually buy XYZ dog food. A giant size bag regularly sells for $5.71. It is **not** on sale. If both brands are equally good, should you buy Big Boy Dog Food on sale or XYZ Dog Food at the regular price?

8. The label on a bottle of hair conditioner says **Mail in enclosed coupon and receive a $.50 refund**. The hair conditioner regularly sells for $2.98. Considering the refund, how much will the hair conditioner cost?

9. The label on a large bottle of juice says **Mail in enclosed coupon and receive a $.75 refund**. The juice regularly sells for $2.25. Considering the refund, how much will the bottle of juice cost?

10. A giant size box of laundry detergent has these words on it: **Price marked is 25¢ off regular price**. The price on the box is $7.22. What is the regular price?

RESOURCE 9: THE BETTER BUY

Use your calculator in order to determine unit prices and the better buy in the purchase situations described.

EXERCISE A:

Suppose you have gone to the grocery store to buy peanut butter and apple juice. Peanut butter is available at $4.97 for a 1 kg container or $3.49 for a 750 g container. Apple juice is priced at $1.59 for a 1.5 L can or 49¢ for a 350 mL can.

Complete the following tables to determine unit prices and the quantity that is the better buy.

Peanut Butter

QUANTITY	PRICE	COST/kg
1 kg	$4.97	
750 g	$3.49	

Apple Juice

QUANTITY	PRICE	COST/L
1.5 L	$1.59	
350 mL	$.49	

EXERCISE B:

Select four different food items. Consult food advertisements in newspapers or visit grocery stores to obtain prices of various sizes and various brands of the food items you have selected.

Complete a table like the one shown for each food item and determine which is the best buy for that particular item.

FOOD ITEM: ____________________				
STORE	BRAND	SIZE	COST	UNIT PRICE

The best buy is ______________________________ because ____________________

____.

Factors other than price that I should consider in buying this item are: ____________________

____.

RESOURCE 10: UNIT PRICING BY MASS AND CAPACITY

In order to compare the price of products sold in different sizes or containers, it is often necessary to find the price for some common unit of the product (i.e., unit price). This requires that we recognize and understand the units of mass and capacity that are frequently used to measure these products. These exercises will provide practice in working with some of the more frequently used units of mass and capacity used to price familiar products.

A. Pricing By Mass

Collect the following food items. Estimate the mass in grams of each item. Then weigh each item to find the actual mass.

FOOD ITEMS	ESTIMATED MASS (g)	ACTUAL MASS (g)
Potato		
Slice of bread		
Boiled egg		
Package of juice crystals		
Chocolate chip cookie		
Weiner		
Head of cabbage		

1. Approximately how many potatoes weigh 1 kg?
2. If potatoes are priced at 40¢/kg, how much would 500 g of potatoes cost? 250 g?
3. Approximately how many weiners weigh 1 kg?
4. If weiners are priced at $2.19/kg, how much does one weiner cost?
5. Approximately how many chocolate chip cookies weigh 500 g?
6. If chocolate chip cookies are priced at $3.49 for 500 g, how much does one cookie cost?

B. Pricing By Capacity

Collect the following food items. Estimate the capacity in millilitres of each item. Then measure each item to find the actual capacity.

FOOD ITEMS	ESTIMATED CAPACITY (mL)	ACTUAL CAPACITY (mL)
Small can of juice		
Can of soup		
Carton of ice cream		
Bottle of ketchup		
Container of margarine		
Large soft drink bottle		
Carton of cereal cream		

1. If ice cream is $2.50/L, how much would 500 mL of this ice cream cost? 250 mL?
2. How many bowls (with a capacity of 150 mL) of ice cream could one obtain from a 1 litre carton of ice cream?
3. If the large bottle of soft drink costs $1.37, what is the cost of the soft drink per mL? Per litre?
4. A 750 mL bottle of ketchup is advertised for $3.95. What is cost per serving of ketchup, if an average serving is 50 mL?
5. If it takes 25 mL of margarine to cover one slice of bread, how many slices of bread can be prepared with this container of margarine?
6. How many cans of juice similar to the one examined would it take to make a litre of the juice?
7. If each small can of juice is 39¢, what would be the cost of a litre of this juice purchased in small cans?

RESOURCE 11: SALE PRICES

Save
30%!
W.G.
Jeans
Here and Now in
Popular Price.

1. Calculate the sale price of each of the following pairs of jeans:

a) $32.95 ________________

b) $28.95 ________________

c) $24.50 ________________

d) $36.80 ________________

e) $25.89 ________________

f) $31.24 ________________

2. Find other advertisements in the newspaper for clothing offered at a discount. What is the percent discount and sale price for each item that is advertised? Mount your advertisements on the reverse side of this page and show your calculations.

____

____

____

____

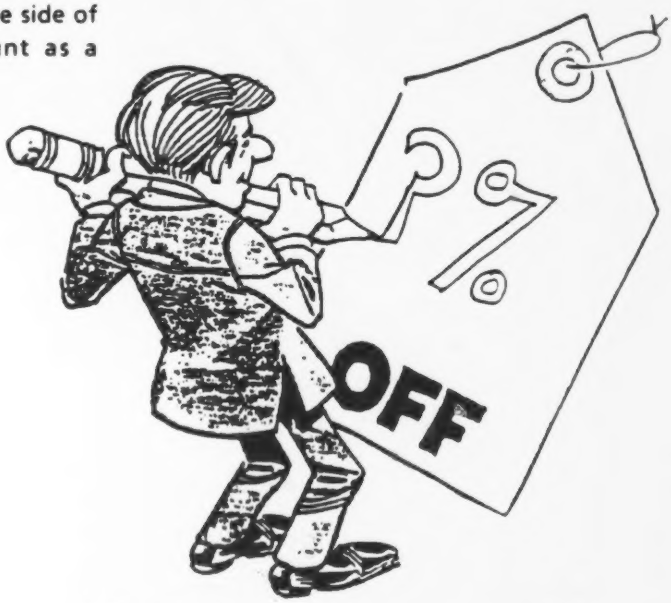
____

Save on...
MEN'S SUITS

From *That Figures, A Math Resource Package for Intermediate Grades* by Marilyn Hintz and Jerry Ziegler. Copyright 1981 by Kitchener-Waterloo Record Ltd. Reprinted by permission.

RESOURCE 12: WHAT IS THE PERCENT DISCOUNT?

Select several display advertisements which offer a given amount off the regular price. Clip and mount the ads to the reverse side of this page. Calculate each discount as a percent.



Original Price	Discount	Percent Discount

From *That Figures, A Math Resource Package for Intermediate Grades* by Marilyn Hintz and Jerry Ziegler. Copyright 1981 by Kitchener-Waterloo Record Ltd. Reprinted by permission.

RESOURCE 13: INSURING YOUR VEHICLE

In Alberta, it is against the law to operate an uninsured vehicle. If you plan to drive a car, you must be sure that it is insured.

LIABILITY INSURANCE

Every vehicle operator in Alberta must have liability insurance. This kind of insurance provides financial protection if you cause injury to another person or their property.

When you buy liability insurance, you must decide on the amount of money for which you wish to be insured. Choose this amount with care, as it represents the greatest amount of money that the insurance company will pay if you have an accident. If damages resulting from an accident exceed this amount, then you must pay the difference. It is worthwhile to carry more liability insurance than you "think" you may need. The cost of being insured for one million dollars is not that much greater than the cost of being insured for five hundred thousand dollars.

COLLISION INSURANCE

Although collision insurance is not required by law, many people have this kind of insurance. If your car needs repairs because of an accident you caused, collision insurance will help pay for those repairs.

In most collision insurance policies, part of the cost of repairs is "deductible" and will have to be paid by you. Some collision policies are $50.00 deductible. This means that you will have to pay for the first $50.00 of damages done to your car. Other collision insurance policies are as much as $500.00 deductible.

COMPREHENSIVE INSURANCE

Another kind of insurance that you may wish to have for your car is comprehensive insurance. This kind of insurance covers your car against damage not caused by a traffic accident. Fire, flood, hail and flying rocks are common causes of damage that may be repaired through comprehensive insurance.

SHOPPING FOR VEHICLE INSURANCE

Two common methods of buying vehicle insurance are:

- buying directly from an insurance company
- buying from an insurance broker, who sells insurance for a number of insurance companies.

It is worthwhile to compare the cost of insurance with several different insurance companies and/or insurance brokers. Different companies may charge different amounts for the same kind of insurance. While cost is an important consideration when shopping for vehicle insurance, one should also consider the reputation of the company in providing prompt and reliable service to policy holders.

RESOURCE 14: AUTOMOBILE INSURANCE RATES

The tables simulated below indicate the cost of various types of automobile insurance sold by the ABC Insurance Company. The premiums are payable annually, and represent the cost of insuring a male driver between the ages of 16 and 25 through this company.

ANNUAL PREMIUMS FOR BODILY INJURY AND PROPERTY DAMAGE LIABILITY INSURANCE					
Driving Record*	Liability Limits				
	$200 000	$300 000	$500 000	$1 000 000	$2 000 000
6	260	268	277	292	318
5	318	329	340	358	390
4	421	435	449	474	515
3	480	506	522	551	599
2	612	633	653	688	749
1	666	688	710	749	815
0	833	861	888	936	1019

ANNUAL PREMIUMS FOR COLLISION INSURANCE																
Driving Record*	Car Rating Group:**															
	3		4		5		6		7		8		9		10	
	Deductible:															
	$100	$250	$100	$250	$100	$250	$100	$250	$100	$250	$100	$250	$100	$250	$100	$250
6	120	102	138	117	159	135	186	158	219	186	259	220	325	277	390	332
5	145	123	167	142	192	163	225	192	265	226	314	267	394	335	472	401
4	181	154	208	177	239	203	281	239	331	281	391	332	491	417	589	500
3	210	179	242	206	278	236	327	278	385	327	455	287	571	485	685	582
2	253	215	290	247	334	284	392	333	462	392	546	464	685	582	821	698
1	295	250	339	288	389	331	457	389	538	458	637	541	799	679	958	815
0	337	286	387	3298	445	378	522	444	615	523	728	618	913	776	1095	931
PREMIUMS FOR COMPREHENSIVE INSURANCE																
Deductible:																
$25	49		45		65		77		90		107		133		160	
$50	42		48		55		65		77		90		114		136	

* The "Driving Record" represents the number of years with a clear driving record.

** The "Car Rating" is based upon the value of the car, its damageability, and the cost of repairs for that particular model.

THEME D
MATH IN THE WORKPLACE

SUB-THEMES

- ESTIMATING AND MEASURING
- USING SCALE DRAWINGS AND MODELS

RATIONALE

Many projects undertaken at home and in the workplace require facility in estimation and measurement, as well as an ability to work with scale drawings and models. This theme provides opportunities for students to:

- estimate and measure length, area, volume, capacity, mass, time and angle within the context of everyday and work-related problem situations
- use scale drawings and models in practical situations that involve repair and/or construction.

Measurement and geometry skills developed within this theme are intended to establish a foundation for further investigations in Mathematics 26, where students will develop strategies for designing projects undertaken in their occupational courses and monitoring inventory in work-related situations.

An emphasis on estimation strategies prior to engaging in activities that involve actual measurement will assist students to establish the size of standard units of measure, relative to each other and to real objects. This approach will also enhance student ability to select measurement tools that are appropriate in a variety of situations. Projects that involve the use of scale drawings and models will assist students to apply measurement and geometry skills in practical situations where they must gather, organize and display information about the physical world.

Cooperative planning among teachers will ensure that activities within this theme are relevant to student needs in other subject areas. For example, teachers may wish to plan instructional activities that focus attention on measurement and geometry skills that are used by students in completing projects undertaken in their occupational courses. The local community provides additional opportunities for students to develop learning objectives through relevant activities related to this theme.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed within this theme can be found in:

- Problem Solving
- Situational and Concrete Approaches
- Assessment/Evaluation.

THEMATIC OBJECTIVES

ESTIMATING AND MEASURING

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- select units and tools of measure that are appropriate in everyday and work-related situations
- estimate length, area, volume, capacity, mass, time and angle measure, using strategies appropriate to the situation
- measure length, capacity, mass, time and angles within acceptable degrees of accuracy
- use formulas to measure indirectly the perimeter, area and volume of regular shapes and solids
- read and interpret the calibrations on meters, scales and gauges that are frequently used in everyday and work-related situations
- use appropriate computational skills when
  - converting among units of measure
  - adding, subtracting, multiplying and dividing similar units of measure
- apply perimeter, area and volume concepts in determining the quantity of materials used in simple construction or repair projects.

USING SCALE DRAWINGS AND MODELS

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- identify geometric figures and relationships present in the physical world
- understand how geometry and measurement provide methods for gathering, organizing and displaying information about the physical world

  e.g.,
  - congruent figures are used in producing templates and patterns
  - similar figures are used in producing scale diagrams and models
- interpret scale drawings and scale models, and identify geometric properties associated with the figures and shapes used in these representations
- use the concept of ratio to determine actual measurements from scale drawings and scale models
- develop ability to use effectively a variety of construction tools (e.g., straightedge, ruler, compass, protractor, computer)
- use appropriate tools and techniques to construct
  - basic two- and three-dimensional geometric figures
  - a geometric pattern or design representative of one that might be used in a logo, floor covering, fabric or wall covering
  - a scale drawing or scale model relevant to a project undertaken at home or in an occupational course.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

Students will be expected to:

- use estimation and measurement to solve practical problems in everyday and work-related situations
- select units and tools of measurement that lend a degree of accuracy appropriate to the task
- develop formulas for finding perimeter, area and volume that summarize relationships and patterns discovered through concrete experience
- gather, organize and display information about the physical world by constructing scale drawings and scale models.

USE OF TECHNOLOGY

Students will be expected to:

- use a calculator to perform computations when
  - solving perimeter, area and volume problems
  - determining actual measurements from a scale drawing or model
- develop an understanding of the graphic capabilities of computer technology, and how computers can be used to create geometric patterns and prepare scale drawings
- use prepared computer software as required to
  - maintain/reinforce basic concepts and skills in ratio, proportion, measurement and geometry
  - develop effective problem-solving strategies
  - construct geometric figures, patterns and scale drawings.

COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform calculations with a calculator on a regular basis
- use mental arithmetic when appropriate to expedite solutions to quantitative problems
- estimate length, area, volume, capacity, mass, time and angle measure in order to:
  - determine quantity/size of materials required for a project
  - check the results of actual measurement
- use estimation to approximate the quantity and cost of materials used in a construction or repair project.

SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

Number Systems and Operations	**Ratio, Proportion and Percent**	**Algebra**	**Geometry and Measurement**	**Data Interpretation and Display**

RATIO, PROPORTION AND PERCENT

Students will be expected to:

- compare two quantities in the same unit by writing ratios in the form a/b, a:b and a to b
- generate equivalent ratios
- calculate rates/unit rates by writing ratios that involve numbers with different units (e.g., 1 cm : 5 m)
- identify proportions as statements about equivalent ratios
- write proportions that describe practical problem situations
- calculate the unknown value in a proportion.

ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions and/or equations that summarize patterns and relationships discovered through concrete experiences
- develop formulas for finding perimeter, area and volume
- use substitution and equation-solving strategies to find the missing element in a formula.

GEOMETRY AND MEASUREMENT

Students will be expected to:

- identify line relationships (i.e., horizontal, vertical, parallel, perpendicular and intersecting)
- identify/sketch/construct basic two- and three-dimensional geometric figures (i.e., rectangle, square, parallelogram, triangle, rectangular solid, cube, cylinder, cone)
- identify the relationship between radius and diameter of a circle
- construct circles, given either radius or diameter
- identify similar and congruent geometric figures
- construct geometric patterns/designs, using a variety of construction tools (i.e., straightedge, ruler, compass, protractor, computer)
- estimate and measure length, capacity, mass, time and angles, using SI units and tools appropriate to the situation
- convert among SI units of length, capacity, mass, and time as required in applications
- identify common SI units of area and volume, and their application in practical situations
- develop and apply strategies for determining
  - the perimeter of figures bounded by line segments
  - the circumference of circles
  - area of rectangles, squares, triangles and circles
  - volume of rectangular solids and cubes.

DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- read and interpret information about tools/materials/procedures displayed in tables, charts and graphs
- construct tables and charts in order to determine the quantity and cost of materials required for a project.

INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with competencies required of students in other subject areas. Projects undertaken in many occupational courses are particularly relevant to this theme, and frequently require students to estimate, measure and interpret technical drawings. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

SCIENCE

Students may:

- use a variety of measuring tools and gauges
- collect data related to an investigation by measuring length, area, volume, capacity, mass, time and/or angle.

SOCIAL STUDIES

Students may:

- apply knowledge of geometric patterns and relationships when reading and interpreting maps
- organize and display information by preparing scale drawings and models.

OCCUPATIONAL COURSES

Students may:

- use a variety of tools, meters and gauges to measure length, area, volume capacity, mass, time and angles
- read and interpret tables, charts and/or graphs in order to complete a procedure or task
- interpret and/or construct technical drawings (e.g., scale drawings, scale models, patterns, templates)
- determine the quantity and cost of materials required to complete a project.

Activities suggested within this theme make reference to projects outlined in the *Student Workbook* for occupational courses in:

- Agricultural Mechanics
- Automotive Services
- Building Services
- Commercial Food Preparation
- Crafts and Arts
- Horticultural Services.

COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer a variety of resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing local community resources, and for developing estimation and measurement skills through situational and concrete learning experiences in the community are provided below.

- Visit local businesses and industries in order to investigate:
  - practical applications of estimation and measurement
  - various tools (e.g., meters, scales, gauges) used in measurement
  - the use of scale drawings, scale models, patterns and templates
  - how computer systems are used on the job.

  Possible community contacts include automotive repair shops, construction sites, food service industries and architectural/drafting firms.

- Construct a scale drawing or scale model of an object/facility within the local community.
  e.g.,
  - a floor plan of a house
  - a technical drawing for a piece of furniture
  - a model of a community park.

- Estimate the quantity of materials required to maintain and/or construct a local community facility.
  e.g.,
  - litres of paint required to restore a local building
  - materials required to fence a yard or park
  - square metres of floor covering required to refinish a floor.

- Invite representatives from local businesses and industries to discuss applications of measurement and geometry in their respective areas of service.
  e.g.,
  - carpenters, painters, tile/carpet layers
  - draftspersons, interior designers, architects
  - graphic artists
  - garment and drapery fabricators
  - food service operators.

ESTIMATING AND MEASURING

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 3 – Measurement and Geometry

SUPPORT RESOURCE

MYN: Unit B – Measurement Skills You Need

OTHER RESOURCES

Applied Mathematics
Chapter 1 - Reading Measurements

Mathways (video program)

Math Wise (video program)

SRA Computer Drill and Instruction, Mathematics, Level D

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Introduce the theme by identifying relevant applications of estimation and measurement in everyday and work-related situations. The video program *Math Wise* (01: Measuring Instruments) describes applications of a variety of measuring tools and illustrates how measurement is useful in practical problem-solving situations.

   Borrow and collect measuring instruments frequently used in the home, and those used in science and each of the occupational courses. Plan activities that will assist students to:

   - identify applications of each measuring instrument
   - use each measuring instrument in a practical situation.

A variety of measuring tools and/or procedures relevant to occupational courses are described in resources at the end of this theme, and include:

- Resource 1: Measurement and Layout Tools Used in Building Services
- Resource 2: Measurement and Marking Tools Used in Agricultural Mechanics
- Resource 3: Metric Measures and Tools Used in Commercial Food Preparation
- Resource 4: Measuring Tools Used in Automotive Services.

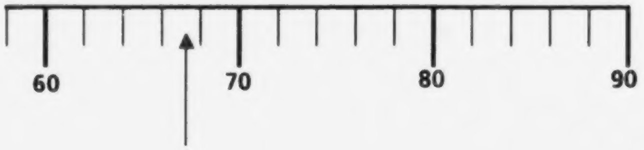
2. Plan activities that provide concrete experience in estimating and measuring length, capacity and mass in practical situations. Encourage students to:

   - develop a familiarity with common SI units of measure through the use of referents
   - use appropriate strategies for estimating length, capacity and mass (e.g., chunking, unitizing)
   - take actual measurements of length, capacity and mass within acceptable degrees of accuracy.

   Appropriate strategies for developing estimation and measurement skills are provided in Situational and Concrete Approaches, "Measurement".

3. Assist students to develop appropriate strategies for reading both decimal and fraction calibrations on familiar measuring instruments. Illustrate different measurement scales on the overhead, and coach students in determining intermediate values on each scale through the process of interpolation.

   CLARIFICATION/EXAMPLE



   The numbers go up by 10. Since there are 5 divisions between 60 and 70, each division must represent 10 ÷ 5 = 2 units.

   Activities that require students to read the scales on a variety of measuring instruments are provided in *Applied Mathematics* (Chapter 1 - Reading Measurements), and in another section of this document (see Situational and Concrete Approaches, "Resource 11: Meters, Scales and Gauges").

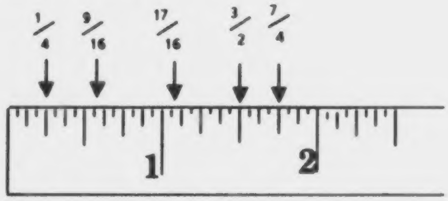
4. Students may be required to use certain Imperial units of measure in their occupational courses. Through cooperative conferencing, identify Imperial units that students may experience in work-related situations, and plan activities in mathematics classes that will assist students to use these units of measure in practical situations.

   Blackline masters that may be useful in developing an understanding of the "inch" unit are provided in Situational and Concrete Approaches, "Resource 2: Using the Inch Unit", and "Resource 11: Meters, Scales and Gauges".

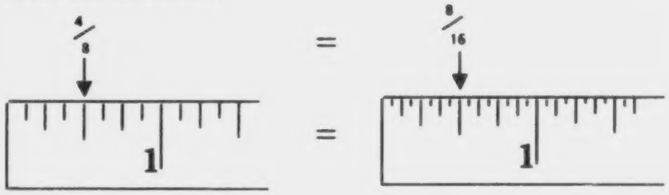
CLARIFICATION/EXAMPLE

USING THE "INCH" RULE

READING MEASUREMENTS

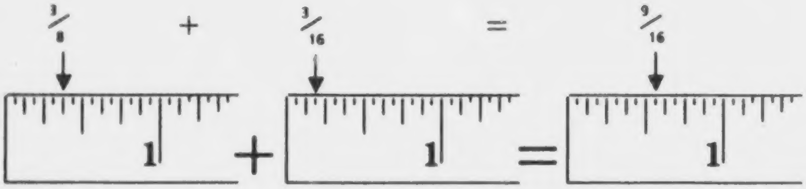


EQUAL MEASUREMENTS



$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$

ADDING MEASUREMENTS



5. Develop strategies for converting among units of measure by asking chains of questions that encourage students to "think through" the conversion process.

CLARIFICATION/EXAMPLE

4500 cm = _______________ m

Sample Question Chain:

- Will I have more or fewer metres than centimetres?
- If I need fewer metres, what operation should I use?
- How do I determine the number by which to divide?
- In what direction and how many places will the decimal move? Why?
- Does the answer seem reasonable?

Once the conversion process is understood, encourage students to develop formal strategies for converting among units of measure (e.g., writing a formula, making a chart, devising a calculator algorithm).

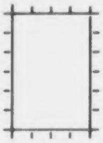
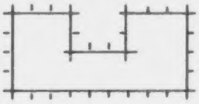
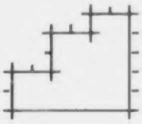
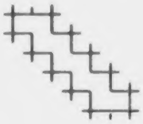
6. Develop perimeter, area and volume concepts through concrete experiences with tactile and manipulative materials (e.g., geoboards, tiles, cubes, geoblocks). These hands-on experiences will improve students' perception of spatial relationships that are inherent in two- and three-dimensional shapes.

Compare the concept of perimeter to "distance around", the concept of area to "surface covered", and the concept of volume to "space occupied".

CLARIFICATION/EXAMPLE

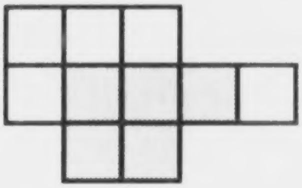
PERIMETER

Find the distance around each pattern. The marks represent distances of 1 metre.

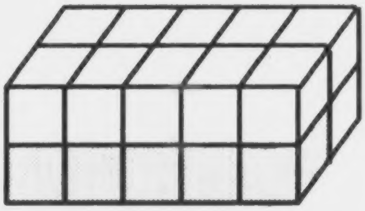
AREA

Fill the shape with tiles or other square units in order to determine surface covered.



VOLUME

How many cubes in one row? How many rows in one layer? How many layers? How many cubes are needed to fill the space inside the box?



Appropriate activities for developing area and volume concepts are provided in Situational and Concrete Approaches, "Resource 9: Understanding Area", and in the video program *Mathways* (Areas, Volumes).

7. Develop perimeter, area and volume formulas as a consequence of patterns and relationships that have been discovered through investigation. Ask questions that will help students to deduce their own strategies for determining perimeter, area and volume based on the results of manipulation and construction.

   The video program *Math Wise* (02: Formulas) illustrates applications of perimeter, area and volume formulas in practical problem-solving situations.

8. Encourage students to make estimates of perimeter, area and volume in practical situations, and to verify their estimates through concrete manipulation, and/or diagramatic representation.

   e.g.,
   - How many metres of fence are needed to go around a garden 6 m long and 4 m wide?
   - How many tiles, each 10 cm by 10 cm, are needed to cover a wall measuring 300 cm by 100 cm?
   - How many cubical boxes, each 3 cm by 3 cm by 3 cm, will fit into a larger box that is 30 cm by 12 cm by 9 cm?

9. Provide opportunities for students to apply perimeter, area and volume concepts in order to determine the quantity and cost of materials used in a construction and/or repair project. A look at sale flyers and newspaper advertisements will prompt students to identify situations where materials are sold by length, area and volume.

   CLARIFICATION/EXAMPLE

Perimeter	Area	Volume
wall panelling fencing trimming	carpeting wallpapering painting tiling	concrete work excavating mixing liquid

   Assist students to organize and plan their work by modelling steps that might be followed in quantifying and costing materials required for the project.

   e.g.,
   - estimating and measuring
   - interpreting/constructing scale drawings
   - applying perimeter, area and volume formulas
   - quantifying materials
   - costing materials.

10. Identify some commercial products whose "directions for use" include reference to area and volume measures.

   e.g.,
   - Paint: "1 litre covers 12 m^2"
   - Fertilizer: "10 kilograms covers 600 m^2"
   - Insecticide: "1 mL (cm^3) per 10 litres of water".

   Display label directions in poster format. Ask students to interpret and explain label directions through diagramatic representation and/or expressive writing activities (see Assessment/Evaluation, "Diagnostic Writing Assignments").

USING SCALE DRAWINGS AND MODELS

RESOURCE CORRELATION

BASIC RESOURCE

CCM: Chapter 3 – Measurement and Geometry

SUPPORT RESOURCES

MYN: Unit B – Measurement Skills You Need

Math Strategies: Problem Solving (computer software)

OTHER RESOURCES

Applied Mathematics
Chapter 5 - Ratio, Scale and Proportion
Chapter 6 - Plane Geometry

Math Wise (video program)

SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives identified in this sub-theme. The activities that follow complement those provided in the student learning resources. In planning appropriate instructional activities, teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning.

1. Assist students to recognize geometric form and relationships present in the physical world by identifying and discussing one-, two- and three-dimensional shapes present in:

    - the natural environment
    - the classroom
    - local architecture
    - community design.

    Summarize the results of investigation, using pictures, sketches or diagrams.

2. Provide opportunities for students to construct one-, two- and three-dimensional figures using traditional construction tools (e.g., compass, protractor, straightedge) and the computer. Throughout construction activities, focus attention on:

    - the properties of basic geometric shapes
    - proper use of construction tools
    - construction techniques
    - strategies for the accurate measure of lines and angles.

Activities that require students to use a compass, protractor, straightedge, T-square, set square and carpenter's square in constructing geometric figures are provided in *Applied Mathematics* (Chapter 6: Plane Geometry).

A list of LOGO commands that might be used to generate geometric figures on the computer is provided in Use of Technology, "LOGO Computer Language".

Patterns for constructing the cylinder, cube and rectangular solid are provided in Situational and Concrete Approaches, "Resource 10: Three-Dimensional Patterns".

3. Ask students to demonstrate their knowledge of geometric figures and construction techniques by creating a geometric pattern or design representative of one that might be used in a logo, floor covering, fabric or wall covering. Encourage students to be creative in the patterns and designs they produce. Display student work in colourful and attractive arrangements.

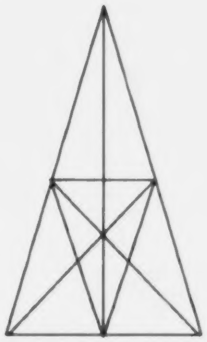
   Sample patterns and designs are provided in Situational and Concrete Approaches, "Resource 3: Geometric Patterns", and "Resource 4: Line Design".

   Additional ideas for projects involving geometric pattern and design are provided at the end of this theme, in "Resource 5: Using Geometric Shapes in Arts and Crafts".

4. Present puzzles and problems that require students to visualize spatial patterns and relationships that are present in two- and three-dimensional figures. Problem-solving activities of this nature will assist students to interpret visual data that is provided in technical drawings.

CLARIFICATION/EXAMPLE

How many triangles can you find?



Sample problems involving tangrams and tessellations are provided in Situational and Concrete Approaches, "Resource 6: The Tangram", and "Resource 7: Tessellations". Appropriate strategies for solving these and other problems are developed in the computer software program *Math Strategies: Problem Solving*.

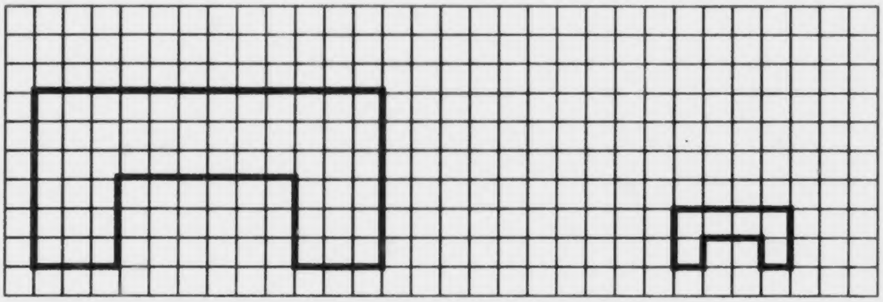
A variety of books containing additional recreational problems and activities in geometry can be obtained from local media centres and libraries.

5. Relate the concept of congruent figures to the use of templates and patterns in project work. Identify projects undertaken at home and in occupational courses that may involve the use of a template or pattern (e.g., sewing, carpentry). Display a variety of templates/patterns that have application in practical situations.

6. Develop an understanding of similar figures by asking students to enlarge or reduce simple geometric figures on grid paper according to established ratios.

CLARIFICATION/EXAMPLE

Sketch a 3:1 reduction of this figure.



Original

Reduction

7. Relate the concept of similar figures to the use of scale drawings and scale models in representing physical objects. Identify situations at home and in occupational courses that involve the use of scale drawings and models. Some practical applications of scale drawings and models in work-related situations are illustrated in:

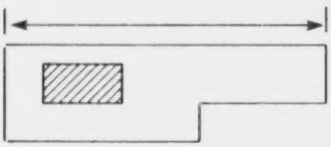
- Resource 6: Patterns and Scale Drawings Used in Agricultural Mechanics
- Resource 7: Technical Drawings Used in Building and Constructing.

Display and discuss a variety of scale drawings/models that have application in everyday and/or work-related situations.

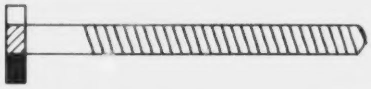
8. Explain how components in a ratio compare the size of a scale drawing to the actual object. Use concrete examples to illustrate how the first number in the ratio represents the measure of the scale drawing, and how the second number represents the measure of the actual object.

CLARIFICATION/EXAMPLE

If the scale is 1:60, what is the actual length of this pattern?



If the scale is 2:1, what is the actual length of this bolt ?



9. Model appropriate strategies for determining actual dimensions of an object from information provided in a scale drawing or model.

CLARIFICATION/EXAMPLE

A 150 metre pool is drawn to a scale of 1 cm = 50 m. How long is the pool on paper?

$$\frac{\text{cm}}{\text{m}} \qquad \frac{1^{\boxed{\times 3}}}{50_{\boxed{\times 3}}} = \frac{?}{150}$$

Strategies that may help students to understand and solve proportion problems are suggested in Situational and Concrete Approaches, "Ratio, Proportion and Percent".

Activities that require students to use ratio and proportion skills in comparing quantities and reading scale drawings are provided in *Applied Mathematics* (Chapter 5: Ratio, Scale and Proportion).

10. Provide opportunities for students to interpret and construct scale drawings and scale models that are used in everyday and/or work-related situations.

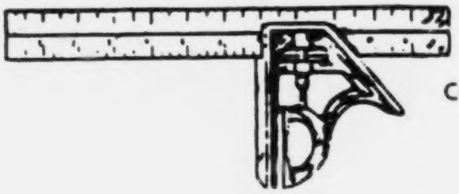
e.g.,
- Interpret and/or prepare scale drawings and models for projects undertaken in other subject areas (e.g., blueprints for a house in construction services, a plot plan for a garden in horticulture, a map used in social studies, a model of a technological device studied in science).
- Ask students to make a scale drawing for one room in their house. Indicate on the drawing, room dimensions and the location of windows, doors, closets and furniture.
- Construct a scale model of a public building or park in the community. Take actual measurements, determine an appropriate scale and construct the model using cardboard and/or other suitable materials.

The video program *Math Wise* (04: Proportions) illustrates applications of ratio and proportion in interpreting and preparing scale drawings.

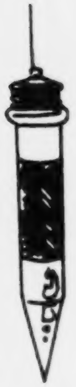
Sample activities that require students to prepare scale drawings are provided in:

- Resource 8: Preparing Scale Drawings at Home
- Resource 9: Preparing Scale Drawings in Horticulture.

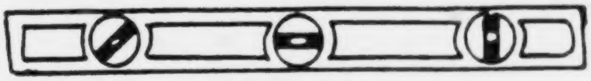
RESOURCE 1: MEASUREMENT AND LAYOUT TOOLS USED IN BUILDING SERVICES



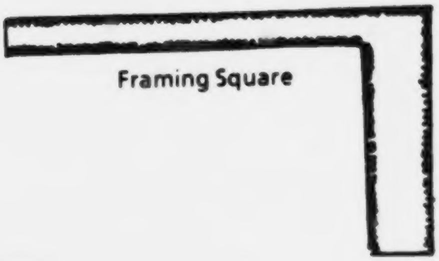
Combination Square



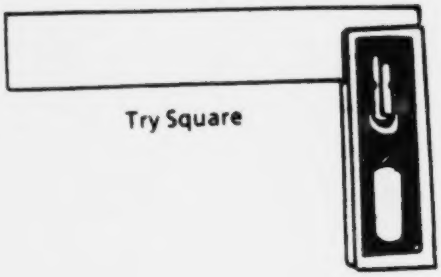
Plumb Bob



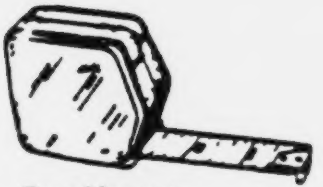
Level



Framing Square



Try Square



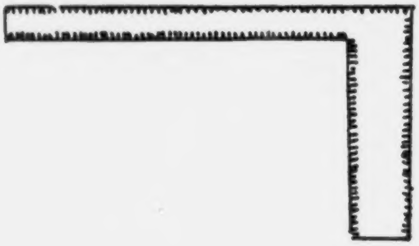
Tape Measure

Scriber

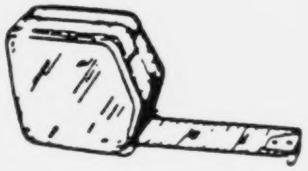
RESOURCE 2: MEASUREMENT AND MARKING TOOLS USED IN AGRICULTURAL MECHANICS

The following tools are frequently used for measuring or marking wood and metal materials. Discuss how to use each of these tools. Place a check beside those tools that you have used in your projects.

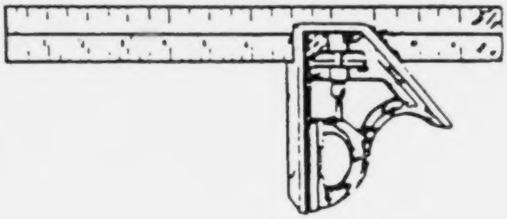
_____ **FRAMING SQUARE**: Used to make a straight edge, to measure, and to align parts.



_____ **STEEL TAPE**: Used to measure lengths.

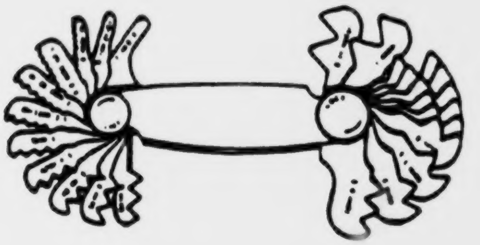


_____ **COMBINATION SQUARE**: Used to make a straight edge, to level, to measure and to align parts.

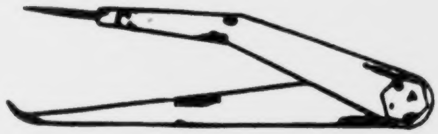


RESOURCE 2: MEASUREMENT AND MARKING TOOLS USED IN AGRICULTURAL MECHANICS (continued)

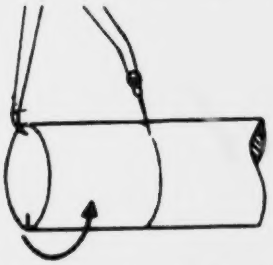
_____ SCREW-PITCH GAUGE: Used to determine the pitch, size, and number of threads.



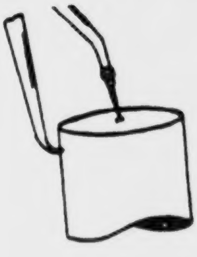
_____ HERMAPHRODITE CALIPER: Used to lay out a line, or to find the centre of a piece of wood or metal.



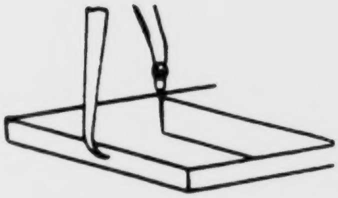
The following diagrams illustrate how this caliper is used.



Marking Lines on Round Surface

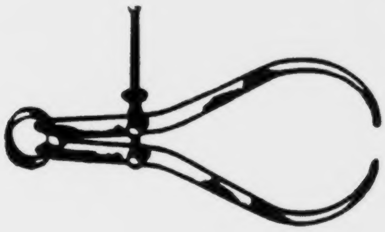


Locating Centers



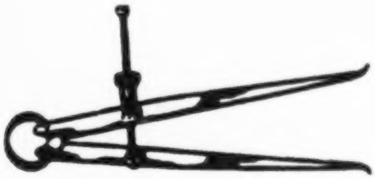
Locating Lines

_____ OUTSIDE CALIPER: Used to measure the outside diameter of an object.

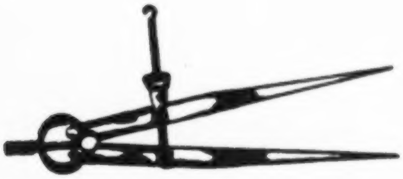


RESOURCE 2: MEASUREMENT AND MARKING TOOLS USED IN AGRICULTURAL MECHANICS (continued)

_____ **INSIDE CALIPER:** Used to measure inside diameters and widths.



_____ **DIVIDER:** Used to lay out circles and arcs, to measure distance between points, and to transfer a measurement from a steel rule.



List below other measurement tools you may be using in your workshop.

From *Vocational Agriculture I.* Copyright 1984 by Oklahoma State Board of Vocational and Technical Education. Reprinted by permission.

RESOURCE 3: METRIC MEASURES AND TOOLS USED IN COMMERCIAL FOOD PREPARATION

Recipes used in food preparation frequently involve the use of metric measures of mass and capacity.

Common metric units of **mass** used in food preparation include:
- gram (g)
- kilogram (kg).

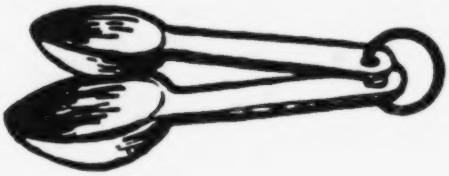
Grams and kilograms are most often used to measure solid products such as flour, butter, cheese and cereals.

Common metric units of **capacity** used in food preparation include:
- millilitre (mL)
- litre (L).

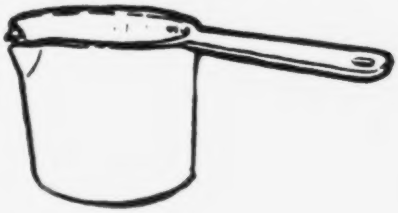
Millilitres and litres are most often used to measure liquid products such as water, milk and ice cream.

Some common metric tools used in baking and other food preparation areas are shown in the illustrations below.

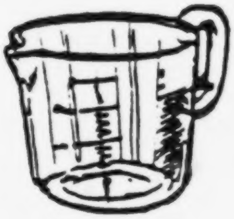
Small Measure



Dry Measure



Liquid Measure



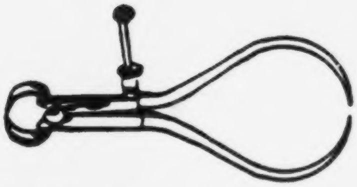
134

RESOURCE 4: MEASURING TOOLS USED IN AUTOMOTIVE SERVICES

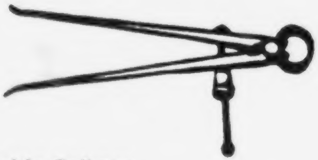
MEASURING TOOLS

A mechanic is constantly taking measurements of things on vehicles. When repairing a car or replacing components, the size of parts (e.g., engine compression, cylinder volume, cable length, tire diameter) must all be measured to ensure that repairs and replacements meet manufacturer's specifications.

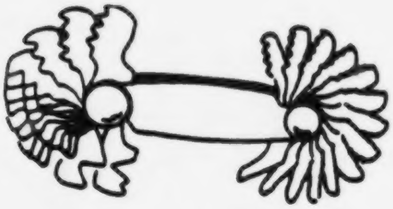
The ability to measure anything accurately depends upon the correct use of measuring tools. Auto workers use a variety of measuring tools, some of which are illustrated below.



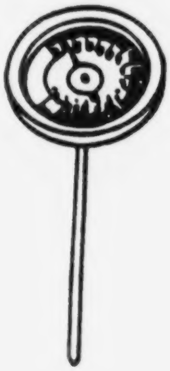
Outside Caliper



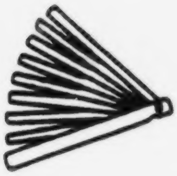
Inside Caliper



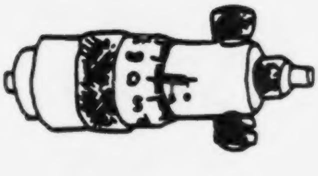
Pitch Gauge



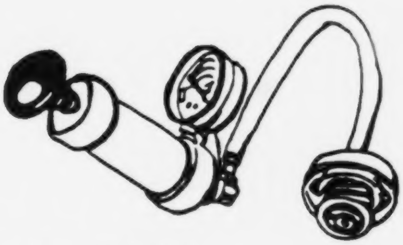
Thermometer



Feeler Gauge



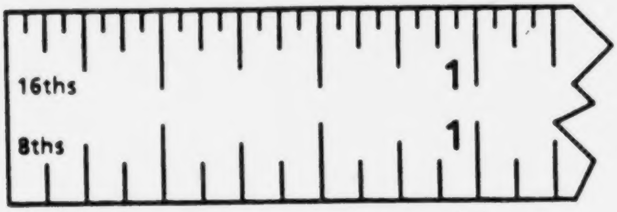
Micrometer



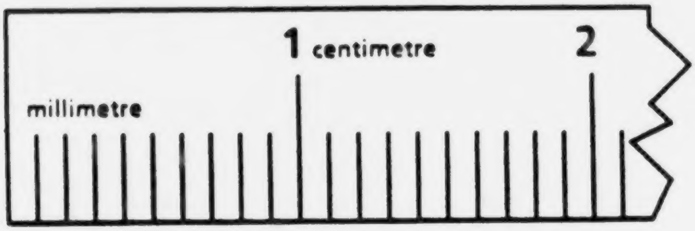
Cooling System Pressure Tester

MEASURING WITH A RULER

Both metric and Imperial measurements are used in automotives. When using an Imperial (fractional) ruler, always reduce fractions to their lowest terms (e.g., 16/64" = 1/4"). A metric ruler measurement should be written in millimetres or centimetres, rather than centimetres and millimetres (e.g., 15 mm or 1.5 cm).



Imperial or Fractional Ruler



Metric Ruler

STUDENT ACTIVITY

1. Draw lines of the following lengths with an Imperial ruler:

   a. 1"
   b. 1½"
   c. 1¾"
   d. 2⅞"
   e. 3 13/16"

136

RESOURCE 4: MEASURING TOOLS USED IN AUTOMOTIVE SERVICES (continued)

2. Draw lines of the following lengths with a metric ruler:
   - a. 10 mm
   - b. 1 cm
   - c. 15 mm
   - d. 2.5 cm
   - e. 3.7 cm

3. Measure the following lines to the nearest inch.
   - a. ______
   - b. _______________
   - c. ______________________
   - d. ______________________________

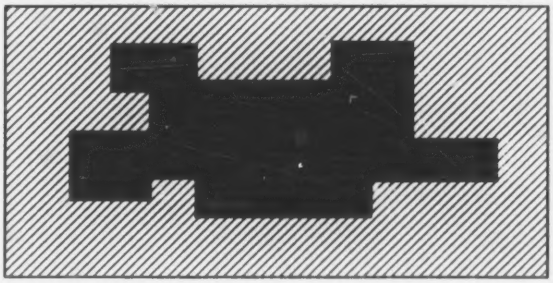
4. Measure the following lines to the nearest 1/4"
   - a. ___________
   - b. _______
   - c. __________________________
   - d. ______________________________

5. Measure the following lines to the nearest centimetre.
   - a. ___________
   - b. _______
   - c. __________________
   - d. _______________

6. Measure the following lines to the nearest millimetre.
   - a. __________________
   - b. _______________
   - c. ____________________
   - d. ______________________________

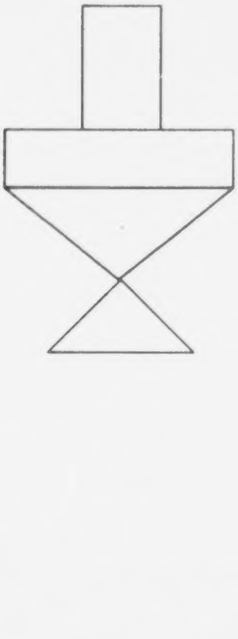
RESOURCE 5: USING GEOMETRIC SHAPES IN ARTS AND CRAFTS

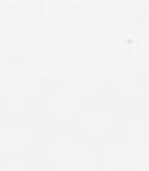
Before drawing any object, it is necessary to understand that the basic element of any design is **SHAPE**. Shape is any two-dimensional area. Wherever one shape exists, there is always a reciprocal; i.e., there are really two shapes in existence, the shape itself (positive space) and the space around the outside of the shape (negative space). The solid black shape can be regarded as the positive shape, while the striped area is the negative shape.



GEOMETRIC SHAPES

Geometric shapes are images such as circles, triangles, squares, rectangles or ovals. New shapes can be made by placing geometric shapes together.

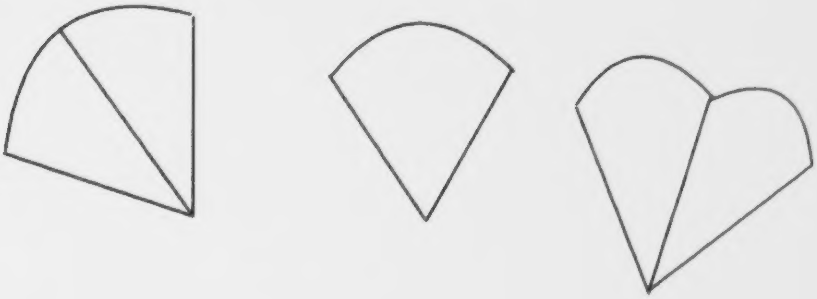




RESOURCE 5: USING GEOMETRIC SHAPES IN ARTS AND CRAFTS (continued)

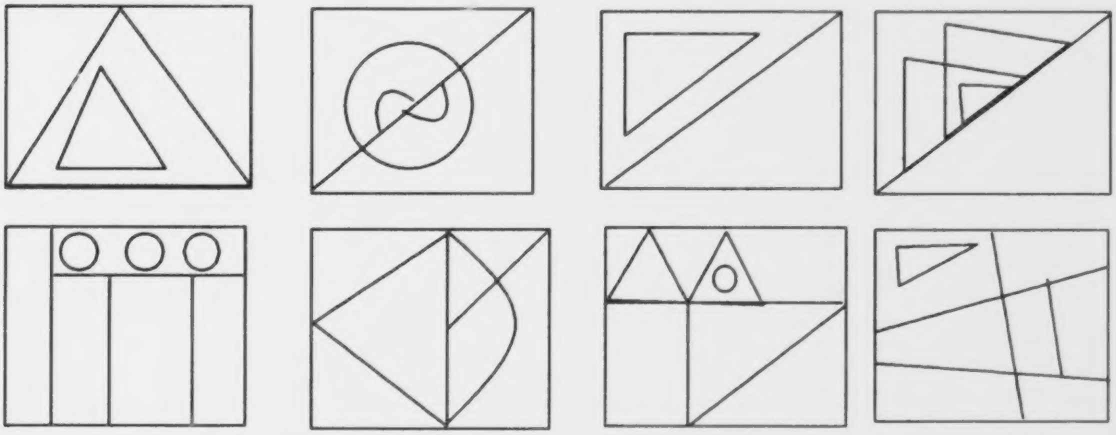
DISSECTING SHAPES

When similar shapes are joined, the image created is of two or more shapes making one design. Dissecting shapes can produce more variety.

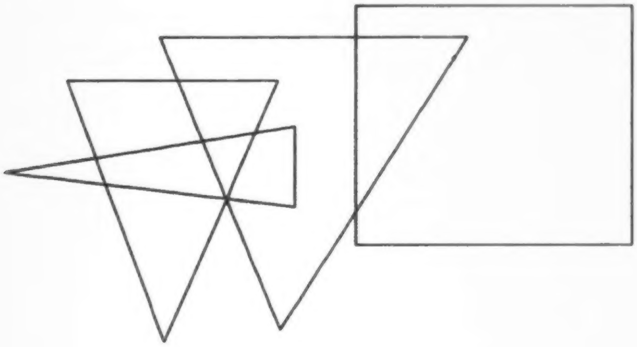


COMBINED SHAPES

Shapes can be placed inside one another to create a range of abstract designs.



Shapes can overlap. Overlapping similar shapes can also achieve interesting designs.



STUDENT ACTIVITY

Draw some original designs consisting of:

a. dissecting shapes
b. shapes placed inside one another
c. overlapping shapes.

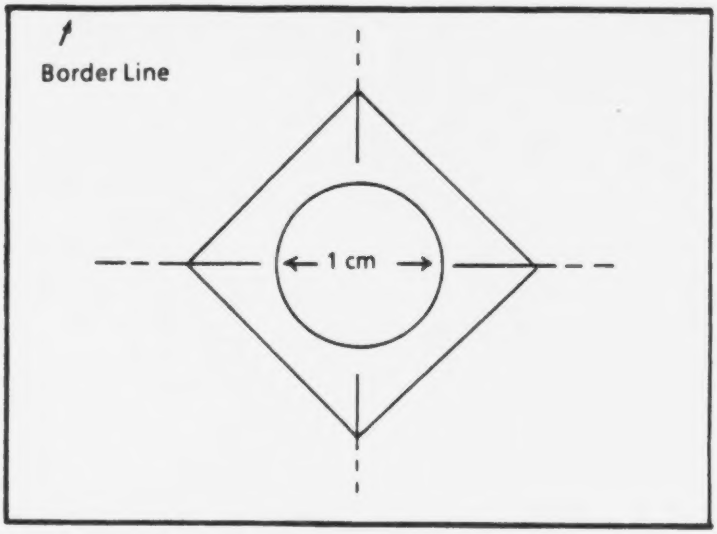
140

RESOURCE 6: PATTERNS AND SCALE DRAWINGS USED IN AGRICULTURAL MECHANICS

When you build or make something, you can follow a pattern or drawing. These drawings tell many things about making a project. They use different lines to describe the project. Some basic types of lines used in drawings are illustrated below.

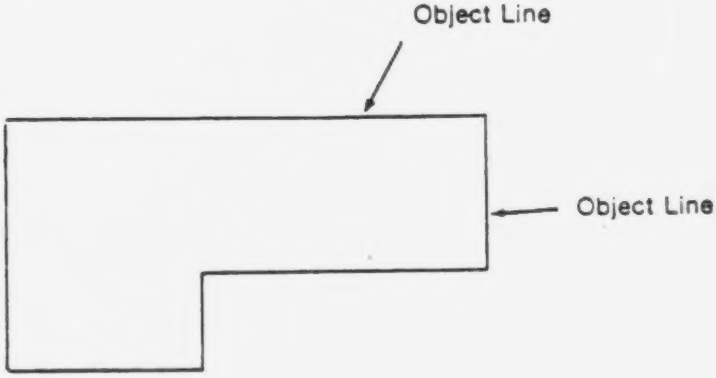
BORDER LINES

Border lines are used to frame the drawing.



OBJECT LINES

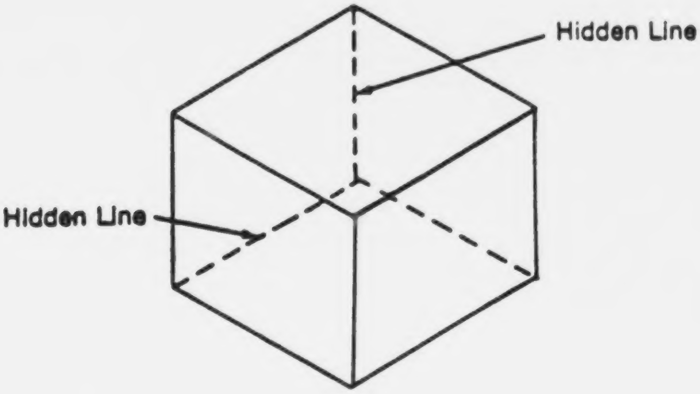
Object lines show the main outline of the object in the drawing.



RESOURCE 6: PATTERNS AND SCALE DRAWINGS USED IN AGRICULTURAL MECHANICS (continued)

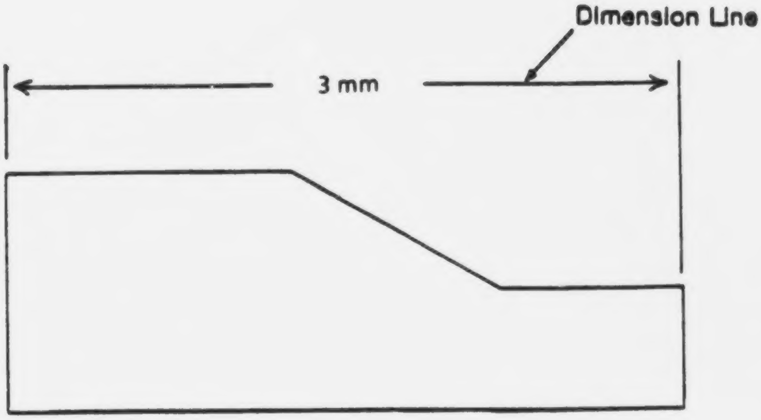
HIDDEN LINES

Hidden lines show lines that are not visible from the view depicted in the drawing.


Hidden Line
Hidden Line

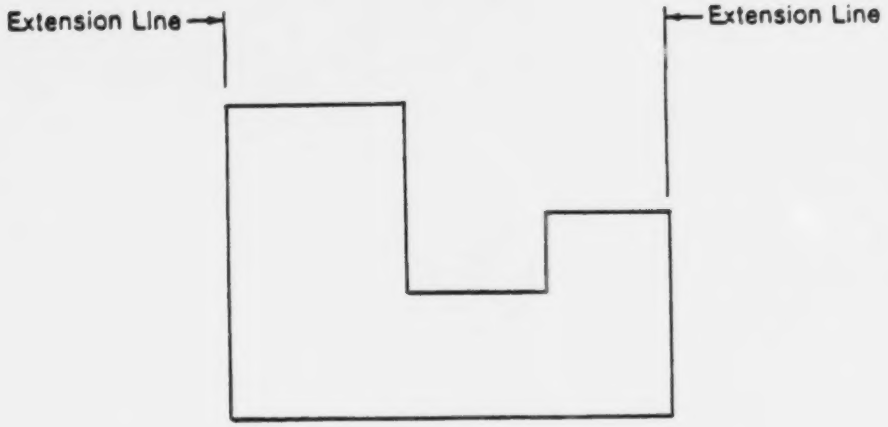
DIMENSION LINES

Dimension lines are used to show the dimensions or size of some feature. They usually end in arrows.


Dimension Line
3 mm

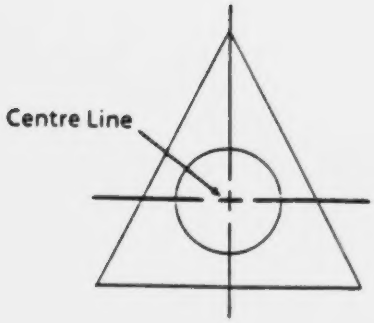
EXTENSION LINES

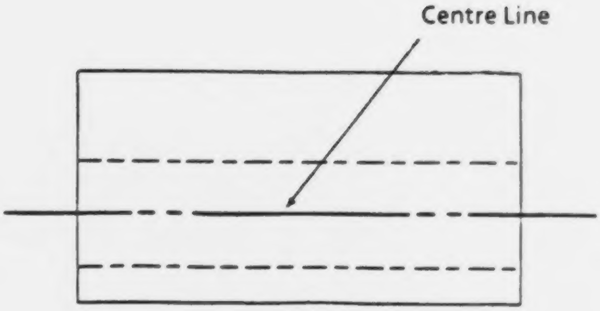
Extension lines are used to show the ends of the dimension lines.


Extension Line
Extension Line

CENTRE LINES

Centre lines show the centre of an object.

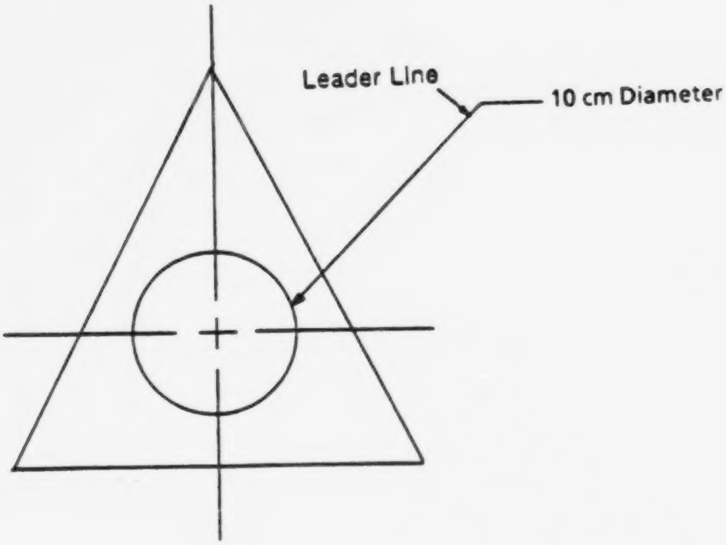

Centre Line


Centre Line

RESOURCE 6: PATTERNS AND SCALE DRAWINGS USED IN AGRICULTURAL MECHANICS (continued)

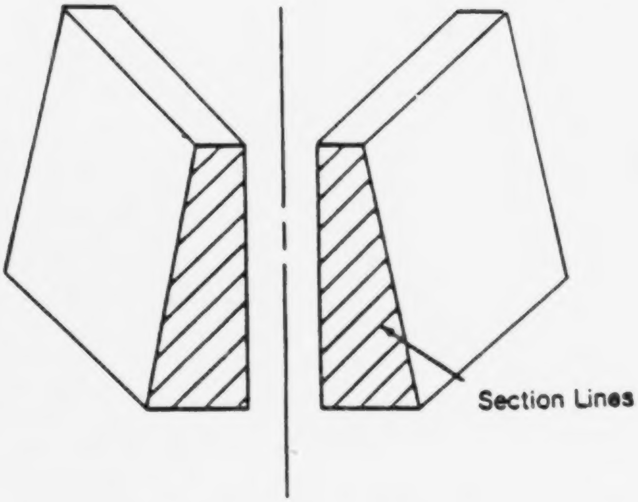
LEADER LINES

Leader lines connect a note to another part of the drawing.


Leader Line
10 cm Diameter

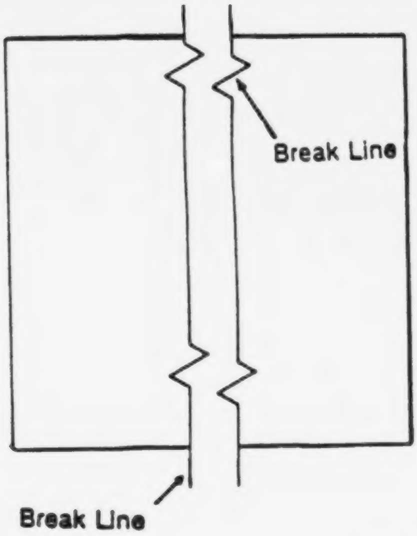
SECTION LINES

Section lines show where an area has been cut away.


Section Lines

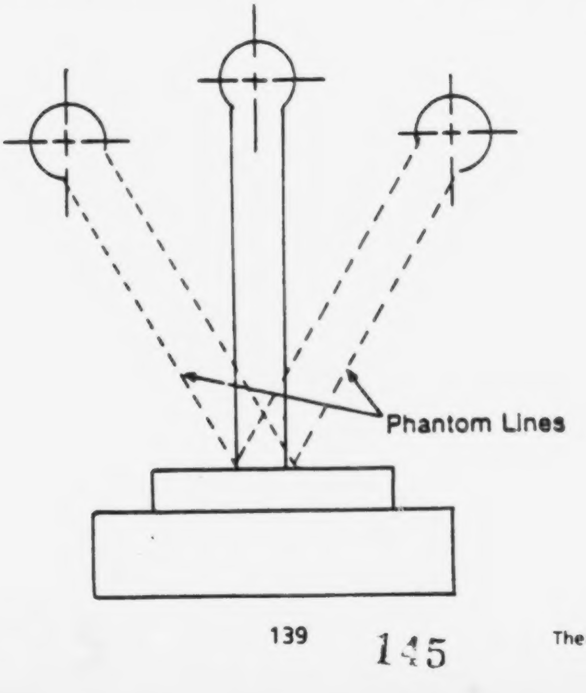
BREAK LINES

Break lines show that an object has not been drawn completely because it was too large. It usually shows the two ends of an object.


Break Line
Break Line

PHANTOM LINES

Phantom lines show an optional position of parts.


Phantom Lines

RESOURCE 7: TECHNICAL DRAWINGS USED IN BUILDING AND CONSTRUCTING

Sketching, drawing, and the ability to understand drawings are important skills for employees in most of the building trades. The 'on site' employee must be able to read and follow the sketches and drawings of architects and engineers in order to know what materials to use and to understand the layout of the project.

The ability to sketch, draw and read simple blueprints or working drawings are useful skills when applying for a job in one of the building trades.

DRAWING TYPES

Different types of drawings commonly used in the building trades include:

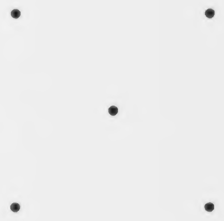
- sketches
- orthographic (3-view) drawings
- isometric drawings
- site plans
- elevation plans
- floor plans.

SKETCHES

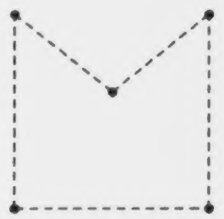
A sketch is the simplest form of drawing. It requires only the use of a pencil and paper. The sketcher may use grid paper if greater accuracy or scaling is desired. Scaling means that one unit of measurement is represented by another unit (e.g., 1 metre on the object = 1 centimetre on the drawing paper).

Sketches may be drawn using the following steps.

STEP 1 Place points at the corners of the object's shape.



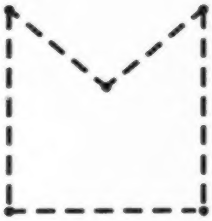
STEP 2 Join the points with light pencil strokes.



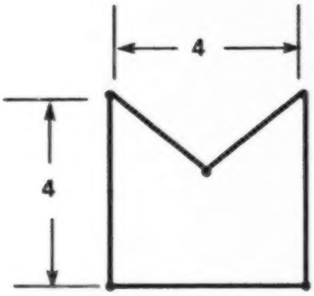
146

RESOURCE 7: TECHNICAL DRAWINGS USED IN BUILDING AND CONSTRUCTING (continued)

STEP 3 Darken the lines.

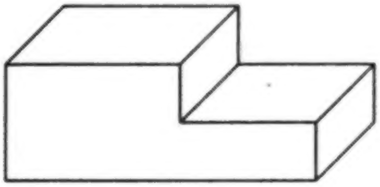


STEP 4 Add any details.



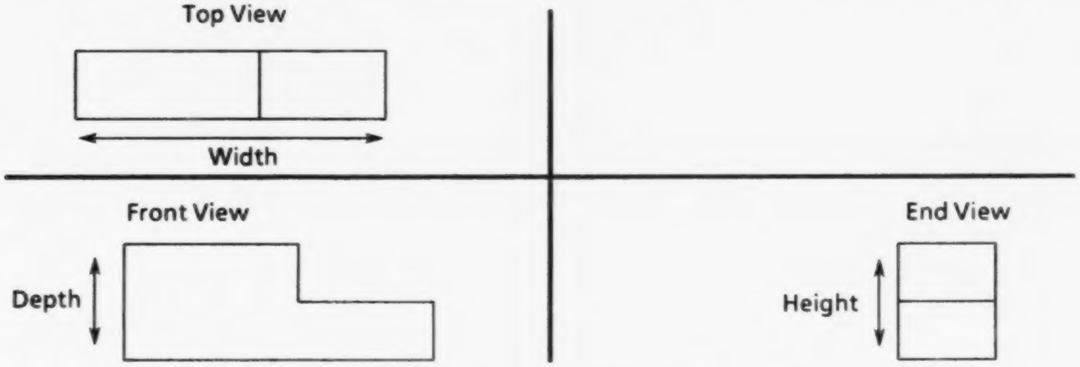
ORTHOGRAPHIC DRAWING

Orthographic drawings are used in the building trades to show one side or view of an object at a time. For example, a three-dimensional step block may look like the diagram below.



RESOURCE 7: TECHNICAL DRAWINGS USED IN BUILDING AND CONSTRUCTING (continued)

An orthographic view of the same object may look like any one of the three diagrams below.



These drawings also show the height, width and depth of the object.

STUDENT ACTIVITY

Sketch an orthographic drawing of some object in the classroom. Share your drawing with others for constructive critique.

Top view

Front View

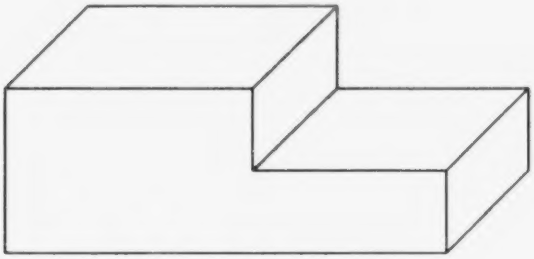
End View

RESOURCE 7: TECHNICAL DRAWINGS USED IN BUILDING AND CONSTRUCTING (continued)

ISOMETRIC DRAWINGS

An isometric drawing is used to show an object as it is. It is a three-dimensional drawing to show where and how the various parts of an object fit together. For example, the drawings that accompany a 'do-it-yourself' kit for assembling a piece of furniture, a model plane or a game, may be isometric drawings.

A simple isometric drawing of a step may look like this:



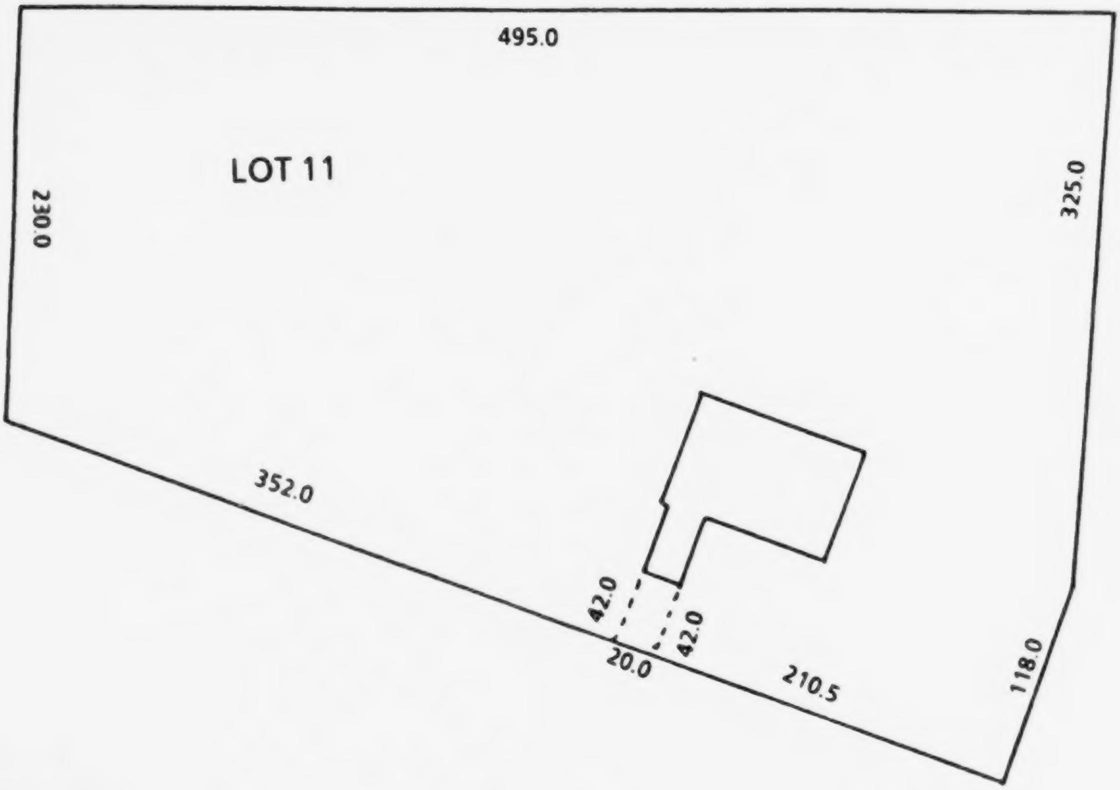
STUDENT ACTIVITY

Sketch an isometric drawing of some object in the classroom. Compare your drawing with those drawn by others. Who has the best "eye" for judging length, height and width?

RESOURCE 7: TECHNICAL DRAWINGS USED IN BUILDING AND CONSTRUCTING (continued)

SITE OR PLOT PLAN

A site or plot plan is used in the building trades to show where, for example, a building is to be constructed on a piece of land called the lot. The site plan is a top view that shows a builder the location of the building and how the building will look in relation to the lot. It also shows how much space can be used for a garden, where trees should be planted, and the angle at which the building can best use sunlight to provide light and possibly heat.



ELEVATION

A drawing or picture that shows the front, rear or side view of a building is called an elevation. When the elevation of a house is drawn it may show:

- floor levels
- windows and doors
- roof line
- types of materials to be used to erect the building.

150

RESOURCE 7: TECHNICAL DRAWINGS USED IN BUILDING AND CONSTRUCTING (continued)

An elevation sketch may look like Figure 1. A drafted, more exact and detailed elevation may look like Figure 2.

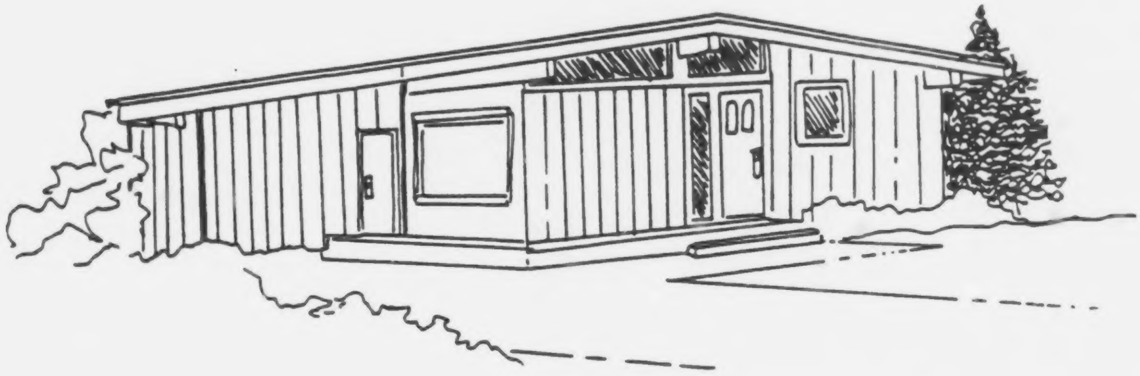


Figure 1. Sketch of Front Elevation

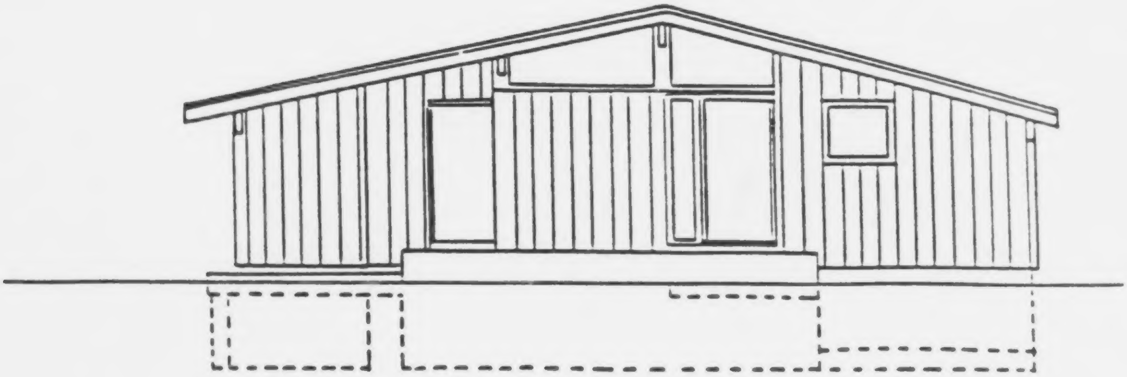


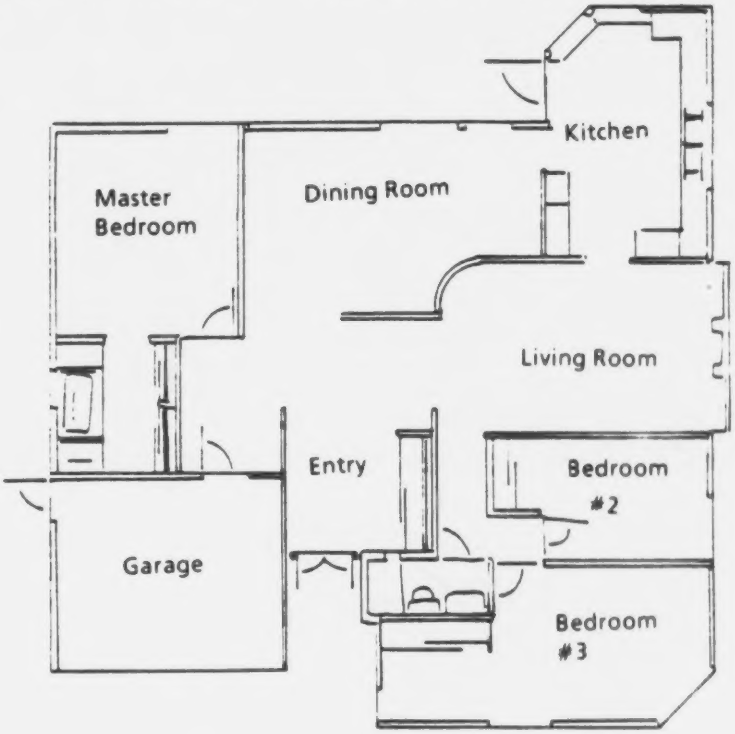
Figure 2. Drafting of Front Elevation of House Shown in Figure 1

FLOOR PLAN

A floor plan shows the size, shape and position of each part of a building. Employees in each of the building trades use floor plans to identify such things as the position of walls, stairs, doors, plumbing and wiring.

A floor plan of a house with three bedrooms, kitchen, living and dining rooms and a garage might look like the one below.

Scale: 1 cm = 8 m


Kitchen
Master Bedroom
Dining Room
Living Room
Entry
Bedroom #2
Garage
Bedroom #3

STUDENT ACTIVITY

Calculate the area for each of the following parts of the house shown on the floor plan:

1. Garage Area = ____________
2. The largest room Area = ____________
3. Bedrooms (1) Area = ____________
   (2) Area = ____________
   (3) Area = ____________
4. The area of the entire house (Excluding the garage) Area = ____________

152

RESOURCE 8: PREPARING SCALE DRAWINGS AT HOME

A. Determine the following measurements of a room in your house (e.g., your bedroom, living room or kitchen).

length of room ____________
width of room ____________
width/location of doorway ____________
width/location of closet doors ____________
depth of closets ____________
width/location of windows ____________

B. Using an appropriate scale, make a scale drawing of this room on centimetre grid paper. Use appropriate conventions to show the location of the door, closets, windows and furniture in the room.

SCALE: 1cm = ______

RESOURCE 9: PREPARING SCALE DRAWINGS IN HORTICULTURE

A garden working design is part of planning. It is a sketch of the location of the different plants in the garden plot. The working plan is drawn to scale on graph paper. For example, a scale of 1:25 means that every 1 cm on the paper equals 25 cm of the actual garden. A garden that is 300 cm by 150 cm would be drawn in a 12 cm by 6 cm rectangle.

The scale drawing would show:

- the size of the garden
- spacing between rows
- length of each row
- space needed for the mature plant
- the north side of the garden
- rows labelled with the names of the vegetables to be grown.

Design a vegetable garden in the space below. Indicate where you would locate different vegetables and show the scale for your drawing.

PROBLEM SOLVING

Learning to solve problems is a major goal of the mathematics program. Today's rapidly changing technological society demands that students are able to apply the mathematical skills learned to new and unfamiliar problem situations in life. Practice in finding answers to routine word problems in mathematics will not provide students with the problem-solving skills they require. Instead, for solving problems, students must acquire strategies that foster the development of critical and creative thinking skills, and be given ample opportunity to apply the strategies and skills they acquire to a wide variety of problem situations in everyday life. The mathematics program must provide opportunities for students to apply knowledge, skill and experience in new and challenging situations where:

- no readily apparent solution or means to the solution is evident
- a person can be temporarily perplexed
- there may be no answer, a single answer, or many answers
- personal and societal factors are involved, as well as mathematical competencies.

Problem solving should not be viewed as an isolated activity. Strategies and skills appropriate to problem solving must be integrated and applied throughout all themes and strands of the curriculum. Teachers should remember that what may be a problem to some students may be only a routine exercise to others, and be prepared to adapt their presentations and expectations to reflect this. Teachers should also model problem-solving behaviour. Students will think problem solving is important if they are encouraged by their teachers, to solve problems and if they see their teachers solving problems.

The framework for problem solving outlined in the Mathematics 16 *Program of Studies/Curriculum Guide* parallels the framework used for solving problems in other subject areas within the Integrated Occupational Program. It suggests a general "plan for action" that can guide students through a broad range of investigative activities (e.g., problem solving, inquiry, decision making, library research). As students become familiar with the framework and discern similarities among the actions taken in different kinds of investigation, they will be more likely to transfer skills learned in specific contexts to other practical situations where methods and outcomes may be uncertain. Teachers should encourage students to recognize how the framework for solving problems in mathematics provides a general plan for action that is also effective in guiding investigations in English, science, social studies and the occupational courses, and how specific skills used within the framework will vary according to the nature of the problem or investigation.

A variety of suggestions and ideas for developing effective problem-solving techniques have been included in this section of the manual, and focus attention on:

- Developing a Desire to Solve Problems
- A Framework for Solving Problems
- Using Algebra in Problem-Solving Situations
- Cooperative Problem Solving
- Monitoring and Evaluating Progress.

Additional information on problem solving can be obtained by referring to Alberta Education's monographs *Problem-Solving Challenge for Mathematics* (1985) and *Problem Solving in Mathematics: Focus for the Future* (1987).

DEVELOPING A DESIRE TO SOLVE PROBLEMS

Critical to the development of skills in problem solving is the attitude with which students approach the task. The development of appropriate attitudes must be nurtured through an atmosphere that fosters flexibility and acceptance. Students must learn to accept and appreciate that being perplexed and unsure is often normal when first encountering a challenging situation, and to take risks in the development of particular problem-solving strategies. The guidelines that follow may assist teachers in planning effective problem-solving activities.

- Create a positive classroom atmosphere that facilitates the development of appropriate attitudes and a desire to solve problems by:
  - encouraging students to be creative, and to generate their own ideas and approaches to problem solving
  - providing adequate support and assistance to students, but not solving their problems for them
  - being willing to accept unconventional solutions, more than one solution, or no solution, where appropriate
  - challenging students to think critically and justify strategies and solutions
  - being enthusiastic and capable of recognizing the students' desire and perseverance to solve problems
  - providing appropriate questions and modelling for students.

- Select problem-solving activities that emerge from real life situations. Ensure that problems are relevant to the interests and experiences of students, and that cognitive demands of the problems match developmental levels of students.

- Modify and vary the approach used so as to ensure interest, participation, and some degree of success by all students. Most students have an inherent desire to accept the challenge provided by a problem.

Teachers can monitor the development of appropriate attitudes and beliefs about problem solving by asking students to respond to statements on an inventory. Student reaction to particular statements about problem solving will indicate to the teacher those attitudes that need further attention and development. Sample statements that might be used in constructing an inventory of problem-solving attitudes are provided on the following page.

Problem-solving attitudes can also be monitored by observing and questioning students as they work in problem-solving situations. Observations can be recorded on individual checklists and rating scales, thus indicating aspects of attitude and performance that require further attention. Sample checklists and rating scales that may be effective in monitoring attitudinal change and development through the school year are provided in the "Assessment/Evaluation" section of this document.

CLARIFICATION/EXAMPLE

Attitude Inventory Items[1]

Mark true or false depending on how each statement describes you in problem-solving situations. There are no right or wrong answers.

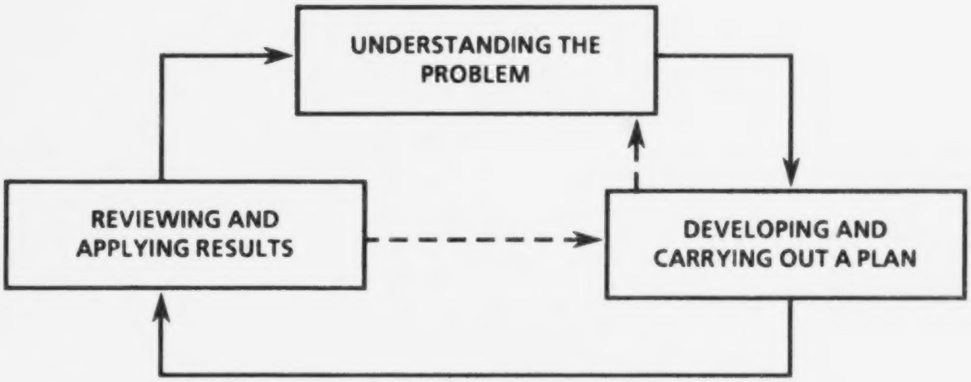
_____ 1. I will put down any answer just to finish a problem.
_____ 2. It is no fun to try to solve problems.
_____ 3. I will try almost any problem.
_____ 4. When I do not get the correct answer right away I give up.
_____ 5. I like to try hard problems.
_____ 6. My ideas about how to solve problems are not as good as other students' ideas.
_____ 7. I can only do problems everyone else can do.
_____ 8. I will not stop working on a problem until I get an answer.
_____ 9. I am sure I can solve most problems.
_____ 10. I will work a long time on a problem.
_____ 11. I am better than many students at solving problems.
_____ 12. I need someone to help me work on problems.
_____ 13. I can solve most hard problems.
_____ 14. There are some problems I will just not try.
_____ 15. I do not like to try problems that are hard to understand.
_____ 16. I will keep working on a problem until I get it right.
_____ 17. I like to try to solve problems.
_____ 18. I give up on problems right away.
_____ 19. Most problems are too hard for me to solve.
_____ 20. I am a good problem solver.

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

A FRAMEWORK FOR SOLVING PROBLEMS

A framework for problem solving is diagrammed below. Although the framework provides an overall structure that can help students to understand the processes used in solving problems, it should not be interpreted as a series of fixed and rigid stages and strategies. There are many ways to approach and solve problems, and the sequence of steps used will depend upon individual problems and individual students. Students may not always go through the steps in the order suggested, nor even use all steps in some problem situations. Nevertheless, an understanding of the steps, as well as individual skills that can be used at each stage of the process, will increase the students' repertoire of strategies that can be brought to bear on a problem.


UNDERSTANDING THE PROBLEM
REVIEWING AND APPLYING RESULTS
DEVELOPING AND CARRYING OUT A PLAN

Teachers can assist students to use the problem-solving process in practical situations by:

- sharing the problem-solving framework with all students. The framework gives structure to the overall process, and suggests specific strategies that might be used in solving problems

- relating the problem-solving framework used in mathematics to the frameworks used for solving problems in science, social studies, English and the occupational courses. Students should recognize how the frameworks are similar in the overall structure they provide for problem solving, and how they may differ in terms of individual strategies appropriate to each stage of problem solving

- keeping the framework and strategies for problem solving flexible and tentative. While useful in the support and structure they provide, students should be encouraged to be creative and experimental in their approach to problem solving.

Performance in problem solving can be enhanced by making students conscious of their thought process (i.e., metacognitive awareness). Students need to become aware of and discuss how they think in order to become more strategic in their learning repertoires, and to monitor their problem-solving efforts (e.g., check their answers when in doubt). Teachers can foster this awareness by thinking out loud as they solve problems, by asking questions, and by having students identify the strategies and processes used when solving problems. Teacher modelling of appropriate steps and strategies for solving problems will provide a structure that facilitates the development of effective problem-solving skills.

Various stages and steps in the problem-solving framework are clarified on the pages that follow. A sample lesson plan illustrating how specific strategies might be used in the problem-solving process is provided in "Resource 1: Sample Lesson Plan for Problem Solving".

UNDERSTANDING THE PROBLEM

During this stage of the problem-solving process, students must be encouraged to think about and interpret the problem situation. Teachers can assist students to focus their attention on information and conditions set in the problem by asking appropriate chains of questions. Teachers can also model and explicitly teach strategies that may be used by students in developing an understanding of a problem situation.

CLARIFICATION/EXAMPLE

Strategies for "Understanding the Problem":

- reading the problem several times
- asking questions
- identifying key words and their meanings
- looking for patterns
- identifying wanted, given, and needed information
- identifying extraneous information
- internalizing the problem by restating in one's own words or by visualizing the problem
- drawing pictures/diagrams
- using concrete manipulatives
- interpreting pictures/charts/graphs
- relating the problem to other problems previously encountered
- simulating or modelling the problem situation
- considering alternative interpretations of the problem
- referring to other resources to clarify the meaning of the problem
- determining if there are hidden assumptions which contain information necessary to the solution of the problem.

Teachers can assist students to understand problem situations by:

- having discussions that focus on understanding the problem
  e.g.,
  - What is the question? What do we need to find?
  - What are the conditions/variables in the problem?
  - What data do we need?
- asking students to explain problems in their own words, or through the use of pictures and diagrams
- reminding students of similar problems
- using coloured markers to highlight important words, phrases or data in the problem
- recording data provided in the form of a list.

DEVELOPING AND CARRYING OUT A PLAN

In this stage, students should plan strategies and then actually use these strategies to solve the problem. As students may lack the "strategic repertoire" required to develop a problem-solving plan, it may be necessary to teach various strategies appropriate to specific problem situations explicitly. Emphasize that there are often strategies other than computation that can be used effectively to solve the problem. Once appropriate strategies have been planned, the student simply "carries out the plan" to arrive at a solution.

CLARIFICATION/EXAMPLE

Strategies for "Developing and Carrying Out a Plan":
- guessing and checking the result (thus improving the guess)
- using logic or reason
- choosing and sequencing the operations needed
- sorting and classifying information
- applying selected strategies
- presenting ideas clearly
- selecting appropriate calculating/measuring devices and methods
- visualizing the problem
- acting out or simulating the problem
- applying patterns
- estimating the answer
- documenting the process used
- working with care
- working in a group situation where ideas are shared
- visualizing the problem
- speaking to self with positive statements (e.g., "I can solve this")
- using a simpler problem (making an analogy)
- identifying factors relevant to the problem
- collecting and organizing data into diagrams, number lines, charts, tables, pictures, graphs or models
- experimenting through the use of manipulatives
- breaking the problem down into smaller parts
- formulating an equation
- recognizing limits and eliminating possibilities
- identifying and applying relationships
- constructing flow charts
- working through the problem backwards
- examining the problem from varying perspectives/points of view.

Teachers can assist students to develop and carry out their problem-solving plans by:

- suggesting a solution strategy
- giving the start of a solution or strategy, and then asking students to complete the solution in order to solve the problem
- giving direct instruction/practice with particular solution strategies
- discussing possible solution strategies
  e.g., – asking students to suggest reasons why they believe particular strategies might work
  – asking what action in the problem suggests a particular operation
- reminding students of similar problems
- providing a one-step or multiple-step problem without numbers, and asking students to identify the operations used in finding a solution
- providing a completed solution to a problem (e.g., a number sentence, organized list, or picture), and asking students to create a problem that would fit the solution.

REVIEWING AND APPLYING RESULTS

This stage encourages students to assess the effectiveness of their solution, and to consider the accuracy of their results. Students should be encouraged to relate answers to the question in the problem in order to verify that the problem has indeed been solved. By evaluating strategies that have been used, students will become aware of their appropriateness and of other strategies that might also be used. Through discussion and reflection, encourage students to generalize and apply their results to related situations.

CLARIFICATION/EXAMPLE

Strategies for "Reviewing and Applying Results":

- stating an answer to the problem
- restating the problem with the answer
- explaining the answer in oral/written form
- determining if the answer is reasonable
- discussing the process used with others
- suggesting other ways of solving the problem
- checking the answer
- considering the possibility of other answers/solutions
- making and solving similar problems
- generalizing the solution and applying the process used to other problems
- creating and writing routine/non-routine problems.

Teachers can assist students to review and assess the effectiveness of their problem-solving efforts by:

- asking students to explain why they chose particular solution strategies
- illustrating alternative strategies that might be used to solve the problem, and evaluating their usefulness
- discussing incorrect strategies and attempts, and explaining why these strategies were not appropriate
- using estimation to check the reasonableness of answers obtained
- ensuring that all relevant information in the problem has been used.

USING ALGEBRA IN PROBLEM-SOLVING SITUATIONS

Students may be reluctant to use algebra in problem-solving situations because of limited experience with algebraic concepts and skills. Teachers are encouraged to model the use of algebraic skills on a frequent basis in both formal and informal problem-solving situations. By using symbols to describe patterns and relationships in practical problem situations, students will gradually develop confidence in their ability to use algebra as a problem-solving tool.

USING VARIABLES TO DESCRIBE NUMBER PATTERNS AND RELATIONSHIPS

Before developing formal equation-solving skills, students should establish an understanding of variables and their use in practical situations. Be aware that an understanding of the concept of variables develops slowly. Demonstrate the use of variables in a variety of concrete situations throughout each theme.

Teachers can encourage students to use variables in describing arithmetical patterns and relationships by:

- playing number games
- drawing diagrams
- building tables
- using/modifying computer programs.

CLARIFICATION/EXAMPLE

A Number Game

Ask each student to think of a number. Then instruct students to:
- double the number
- add 15
- subtract 10
- state the new number.

The teacher can determine each student's original number by reversing the process (i.e., subtract 5 from the answer and divide by 2). Ask students to suggest alternative methods of determining the original number. Describe the problem with an algebraic equation (e.g., $2N + 15 - 10 = 2N + 5$). Encourage students to recognize that in solving equations we frequently reverse operations and the order in which they are performed.

Modify the problem so as to involve the use of brackets and the distributive property:
- multiply the number by 2
- add 10
- divide this number in half
- subtract four.

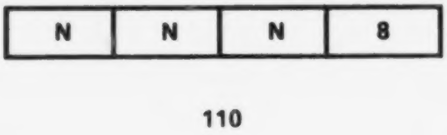
The original number will be one less than the answer.

162

CLARIFICATION/EXAMPLE

Drawing a Diagram

A piece of wood 8 centimetres long is cut off the end of a longer board that is 110 centimetres long. If the remaining board is divided into three equal pieces, how long will each piece be?



Equation: $8 + 3N = 110$

(Additional activities that involve the use of diagrams are provided in Resource 2: Using Diagrams to Solve Problems).

CLARIFICATION/EXAMPLE

Building a Table

If a car is travelling at 60 km/h, determine the distance it has travelled throughout each period of time indicated in the table.

Hours travelled	1	2	3	5	8	10	15	h
Distance travelled								

Encourage students to write a formula that describes the relation illustrated in the table.

e.g., $d = 60 \times h$

CLARIFICATION/EXAMPLE

Using a Computer Program

The computer program illustrated below has been designed to generate a list of perfect squares. Ask students to run and possibly modify this program.

```
10    FOR X = 1 TO 15
20    PRINT X, X^2
30    NEXT X
40    END

RUN

1     1
2     4
3     9
4     16
5     25
•
•
•
```

EVALUATING EXPRESSIONS AND SOLVING EQUATIONS

Early experiences in solving equations should involve the use of "guess and check" strategies. These activities will assist students to develop an understanding of variables and their function as place holders in an equation.

CLARIFICATION/EXAMPLE

Using Guess and Check Strategies to Solve Equations

Find a number that satisfies the equation "$2N+3=11$".

First guess:
$N=1$
$2(1)+3=5$
The number is too small.

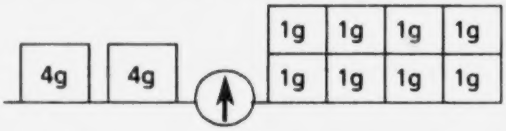
Second guess:
$N=5$
$2(5)+3=13$
The number is too large.

Third guess:
$N=4$
$2(4)+3=11$
The number satisfies the relationship described by the equation.

Following experience using guess and check strategies, students should develop an understanding of more formal strategies used in solving equations. Discuss the concept of "equality" as a statement of balance in number sentences where the left side equals the right side. Use the balance scale as a manipulative, and describe various states of balance on the scale using appropriate number sentences.

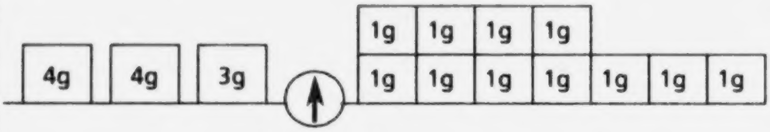
CLARIFICATION/EXAMPLE

Relating Equality to the Balance Scale



$4+4=8$

To maintain balance, whatever is added to/taken away from one side must be added to/ taken away from the other side. Likewise, if one side is increased/decreased through multiplication or division, a similar operation must be performed on the other side of the equation to maintain balance.

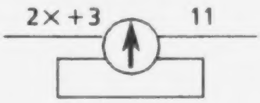


$4+4+3=11$

Relate equation solving to the analogy of balancing a scale. Encourage students to recognize the importance of maintaining balance (equality) in the equation-solving process.

CLARIFICATION/EXAMPLE

If 2× + 3 = 11, then the scale is balanced with 2× + 3 on one side and 11 on the other.



To solve for ×, it must be isolated on one side of the scale. Subtract 3 from both sides. The scale is balanced with 2× on one side and 8 on the other side.

Divide each side of the scale by 2. The scale is balanced with × on one side and 4 on the other side.

X = 4

Another equation-solving strategy involves the analogy of packing/unpacking boxes. This analogy will develop an awareness of the need to reverse operations and the order in which they are performed when solving equations.

CLARIFICATION/EXAMPLE

Packing/Unpacking Boxes

Relate the process of writing an equation to "packing" a box. In solving the equation, the box must be "unpacked".

To solve 2× + 3 = 11, first determine how the box was packed. (i.e., How were the symbols in the equation combined?)

Packing

1. Start by placing an × in the box. What was done to the ×?
2. Multiply it by 2. What was done next?
3. Add 3. How many in total? 2× + 3 or 11.

Now unpack the box to solve for ×. Remind students that when unpacking a box, actions taken to pack the box are performed in the reverse order.

Unpacking

Reverse the packing process as follows:

1. Subtract 3. This yields 8.
2. Divide by 2. The result is 4.

A STRUCTURED APPROACH FOR APPLYING ALGEBRA SKILLS

A structured worksheet is often effective in helping students to cope with the cognitive demands of algebraic representation and equation solving. The structure and organization provided by a worksheet may clarify complexities of a problem, and also prompt students to use appropriate strategies in correct sequence.

CLARIFICATION/EXAMPLE

Sample Format for a Problem-Solving Worksheet

1. Problem-solving goal:

2. What I do not know:

3. What I know:

4. A description of the problem situation in my own words (or a picture of the problem):

5. Algebraic representation of the problem (i.e., an equation):

6. Solving the equation:

7. Solution:

8. Is my solution reasonable?
   (i.e., estimate and check the answer)

9. Does the solution achieve my problem-solving goal?

COOPERATIVE PROBLEM SOLVING

Problem solving provides opportunities for students to develop strategies and skills by working in cooperative learning situations. Group work often provides students with a less threatening environment, where they may be more willing to take the risks associated with problem solving. Students participating in a group problem-solving activity can learn new strategies from others, and refine their own problem-solving skills.

Skills often best learned in group settings include the ability to:

- clarify one's own ideas
- consider alternative ideas and approaches
- compare and assess alternatives.

CLARIFICATION/EXAMPLE

A Paired Problem-Solving Strategy[1]
Students are divided into pairs (problem solver, recorder) in order to work together in solving a problem. The use of a "thinking-aloud" procedure allows the student to see how their partner thinks and solves problems. Thinking steps are thus open to view and can be observed and communicated. The procedure used is as follows:

1. One member (the problem solver) "thinks aloud" while solving a given problem.
2. The other member (the recorder) listens carefully, noting the steps taken in the solving of the problem. At the end of this "think aloud" procedure, the recorder may ask clarification questions of the problem solver and/or may point out errors made in the problem-solving process.
3. Roles are reversed, repeating the same problem.
4. The modification/extension of both strategies are discussed with each person (or both if agreement is reached) documenting the best "modified" strategy.

This strategy could be expanded to include three people by subdividing the recorders' role into recorder and questioner. The three roles would rotate.

Interpersonal skills can also be enhanced through cooperative learning experiences. As students work in group settings, problems in social interaction may arise. A strategy for systematically analyzing a social problem is provided in Resource 3: Social Problem-Solving Strategy. This strategy helps students to identify:

- reasons for the difficulty/conflict
- strategies that may avoid the difficulty/conflict another time.

Teacher modelling and student use of this strategy may assist students to gain confidence in their ability to interact and communicate with one another effectively.

---

[1] Adapted from *SPELT: A Strategies Programme for Effective Learning/Thinking: Inservice Edition,* SPELT International Ltd., p. 197. Copyright 1987 by R. Mulcahy, K. Marfo, D. Peat and J. Andrews. Reprinted by permission.

Teachers can facilitate cooperative problem solving by establishing a "problem-solving corner" in their classrooms. This area of the classroom can be stocked with a variety of problems, puzzles and manipulative materials. Encourage students to contribute interesting problems and puzzles to the problem corner. By displaying a "daily puzzle" or "problem of the week", students can be motivated to use the corner on a regular basis as time permits. Additional ideas that may be useful in establishing a problem-solving corner are provided in an ensuing section of this manual (see "Situational and Concrete Approaches".)

Sample problems provided in "Resource 4: Problem-Solving Ideas" have been selected on the basis of their cognitive demand as well as effectiveness in developing problem-solving strategies. While some of the problems are suited only to group settings, many can be solved by students working independently or in small groups. **Teachers are encouraged to develop their own collection of problems/puzzles that are appropriate to curriculum goals and student interest/ability.**

Additional problem-solving activities are suggested in:

- *Problem Solving Challenge for Mathematics* (Alberta Education, 1985)
- *Problem Solving in Mathematics: Focus for the Future* (Alberta Education, 1987)
- *The Arithmetic Teacher* (published by the National Council of Teachers of Mathematics)
- *The Mathematics Teacher* (published by the National Council of Teachers of Mathematics).

Books and kits providing suitable problems and puzzles can be obtained by contacting local library and resource centres.

MONITORING AND EVALUATING PROGRESS

Checklists will enable the teacher to determine the extent to which students are using certain strategies in problem solving. By keeping a class checklist during work on a particular theme, teachers can diagnose both individual progress in the use of various strategies, as well as total group progress for each strategy emphasized.

CLARIFICATION/EXAMPLE

	Identifies Key Words	Draws Pictures/ Diagrams	Uses Calculator	Applies Patterns	Applies Concepts	Asks Questions	Identifies Relationships	Makes Measurements	Estimates Answer	Explains Process	
Mary	I	II	IIII	I	I		I	III		II	15
Tom	I		I		II		I	III	II	I	11
	2	2	5	1	3	0	2	6	2	3	

In the example above, a tally is inserted in the appropriate column each time the teacher notes tangible evidence of the use of a particular strategy. In this instance, Mary is gaining experience in drawing sketches, making measurements, and in using several other strategies; she has shown no evidence of asking questions or estimating answers. The teacher can then plan special lessons to reinforce the strategies that have not been used.

Checklists can also be used to monitor problem solving attitudes and behaviours. By observing and questioning students as they work in problem-solving situations, teachers can make note of various aspects of attitude and behaviour and record these on a checklist. These records will be useful in assessing attitudinal change and development for individual students throughout the school year.

CLARIFICATION/EXAMPLE

Observational Checklist of Problem-solving Attitudes and Behaviours[1]

______ 1. Likes to solve problems
______ 2. Works cooperatively with others in the group.
______ 3. Contributes ideas to group problem solving.
______ 4. Perseveres – sticks with a problem.
______ 5. Tries to understand what a problem is about.
______ 6. Can deal with data in solving problems.
______ 7. Thinks about which strategies might help.
______ 8. Is flexible – tries different strategies if needed.
______ 9. Checks solutions.
______ 10. Can describe or analyze a solution.

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

Students need to recognize the importance of reflecting on the strategies they use, on what they have done, and on what they still need to do. Encourage students to monitor and evaluate their own thinking and progress in problem solving through the use of focus questions that ask them to think back and describe how they solved particular problems.

CLARIFICATION/EXAMPLE

Focus Questions for Monitoring and Evaluating Progress[1]

Use the following questions to help you look back and describe your thinking as you worked toward a solution to the problem.

1. What did you do when you first saw the problem? What were your thoughts?
2. Did you use any problem-solving strategies? Which ones? How did they work out? How did you happen to find a solution?
3. Did you try an approach that didn't work and have to stop and try another approach? How did you feel about this?
4. Did you find a solution to the problem? How did you feel about this?
5. Did you check your answer in any way? Did you feel sure it was correct?
6. How did you feel, in general, about this problem-solving experience?

Students can also be encouraged to reflect upon the thought processes they use by completing a "strategy inventory" that is based on a particular problem-solving experience.

CLARIFICATION/EXAMPLE

Problem-Solving Strategy Inventory[2]

Think about your use of strategies when solving the problem. Check the following that apply.

1. ______ I didn't think about using strategies at all.
2. ______ The idea of using strategies came to my mind, but I didn't think about it much more.
3. ______ I looked at a strategy list, but didn't try a strategy.
4. ______ I looked at a strategy list and picked a strategy, which I tried.
5. ______ I didn't look at a list, but just thought of a strategy to try.
6. ______ I used at least one strategy and it helped me find a solution.

7. I tried the following strategies:
   ______ guess and check
   ______ make a table
   ______ look for a pattern
   ______ make an organized list
   ______ other ______________________________
   ______ solve a simpler problem
   ______ work backward
   ______ draw a picture
   ______ write an equation

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

[2] Ibid.

Problem solving is often difficult to evaluate because it is process oriented. Be sure to evaluate the problem-solving process that is used, and not just the solution. A possible marking scale has been provided below. This scale assigns 0, 1 or 2 points for each stage of problem solving, according to the criteria stated. It should be noted that the marks assigned at any given stage should not be influenced by the marks assigned at other stages of the process.

CLARIFICATION/EXAMPLE

Marking Scale for Problem Solving[1]	
Understanding the Problem	0: Complete misunderstanding of the problem 1: Part of the problem misunderstood or misinterpreted 2: Complete understanding of the problem
Developing and Carrying Out a Plan	0: No attempt, or totally inappropriate plan 1: Partially correct plan based on part of the problem being interpreted correctly 2: Plan could have led to a correct solution if implemented properly
Reviewing and Applying Results	0: No answer or wrong answer based on an inappropriate plan 1: Copying error; computational error; partial answer for a problem with multiple answers 2: Correct answer and correct label for the answer

Students should not be evaluated on their ability to solve problems simply by what they produce on paper. Attitudes and behaviours that may also be taken into account include:

- willingness to attempt a problem
- use of a systematic approach
- use of appropriate strategies
- willingness to try other strategies
- logical justification of strategies and solutions
- perseverance in the task
- confidence in ability to solve problems
- willingness to contribute to group problem-solving activities
- willingness to solicit/accept help from others.

Evaluation procedures should include a variety of techniques, including observation, questioning and interviewing. Checklists, inventories and rating scales provide a useful means of recording student attitudes and behaviours as they relate to problem solving. Assessment and evaluation techniques are further discussed in an ensuing section of this manual (see "Assessment/Evaluation").

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

RESOURCE 1: SAMPLE LESSON PLAN FOR PROBLEM SOLVING

PROBLEM SITUATION

Six people attend a meeting at 6:00 p.m. on June 6. If each person shakes hands with every other person, how many handshakes would take place?

I. UNDERSTANDING THE PROBLEM

1. Have a class discussion about the problem.
    - Underline key words and important information.
    - Cross out extraneous information.
    - Determine if there are hidden assumptions which contain information necessary to the solution of the problem.
2. Ask questions to focus attention on key components of the problem.
    - How many people are in the room?
    - How many people shake hands at once?
    - If I shake your hand, is that the same as you shaking my hand?

II. DEVELOPING AND CARRYING OUT A PLAN

1. Select an appropriate problem-solving strategy. Possibilities include:
    - Using a table or chart.
    - Drawing a diagram.
    - Looking for a pattern.
    - Acting out or simulating the problem.
2. If students experience difficulty, ask questions that will direct thought processes to appropriate strategies and considerations.
    - Once I shake everyone's hand, how many people will you still need to shake hands with?
    - How many handshakes would be needed if there were only two people at the meeting? Three people at the meeting?
3. Provide encouragement and support to students as they apply strategies and work toward a solution to the problem.

III. REVIEWING AND APPLYING RESULTS

1. Restate the problem with the answer.
    - If six people are in a room, it would require 15 handshakes for everyone to shake hands with everyone else.
2. Discuss other ways to solve the problem.
    - Organize data into a table.

Person No.	No. of Handshakes
1	5
2	4
3	3
4	2
5	1
6	0

The solution can be obtained by systematically listing the number of handshakes made by each person in the group. The total number of handshakes will be the sum of the second column.

- Use logic and reason.

Each person must shake hands with five other people.

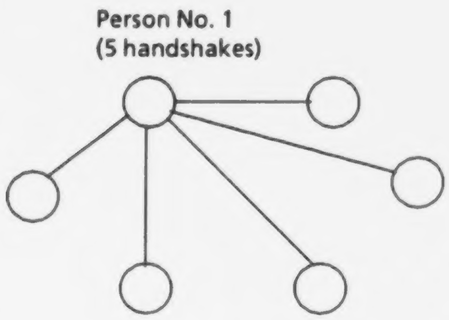
Since there are six people making five handshakes each, there would be a potential total of 6 × 5 or 30 handshakes.

However, each handshake has been counted twice (i.e., if I shake your hand, then you don't need to shake my hand). Therefore, the total number of handshakes will be 30 ÷ 2 or 15.

- Draw a Diagram.

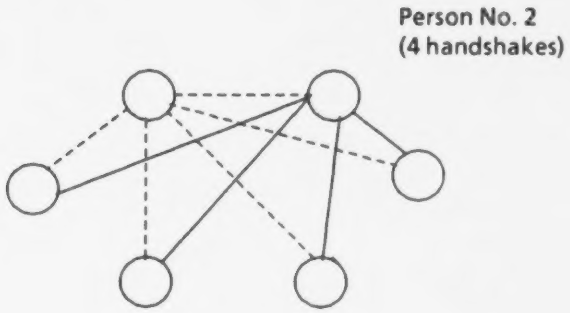
Step One

Draw the six people. Then draw a series of lines that represent the handshakes made by person No. 1.


Person No. 1
(5 handshakes)

Step Two

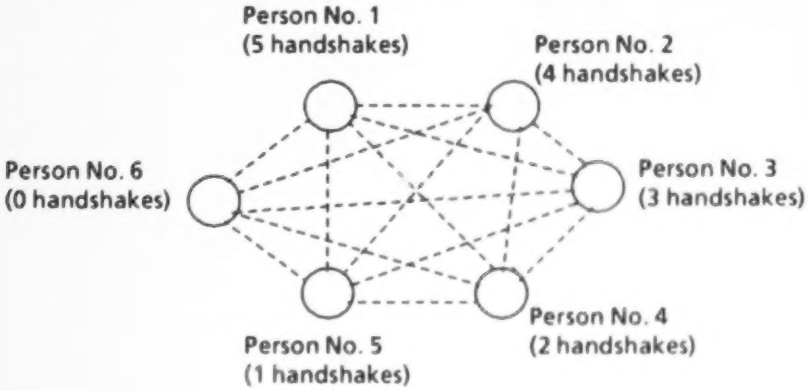
Draw additional lines in the diagram that represent the handshakes made by persons No. 2, No. 3, No. 4 and No. 5.


Person No. 2
(4 handshakes)

(This diagram can be extended to represent the handshakes made by persons No. 3, No. 4 and No. 5.)

Step Three

Notice, in the diagram below, that when person No. 5 has completed his or her handshakes, each person will have greeted every other person with a handshake.


Person No. 1
(5 handshakes)
Person No. 2
(4 handshakes)
Person No. 6
(0 handshakes)
Person No. 3
(3 handshakes)
Person No. 4
(2 handshakes)
Person No. 5
(1 handshakes)

The total number of handshakes, determined by counting the lines, must be 15.

3. Generalize the solution and apply the process used to other problems.

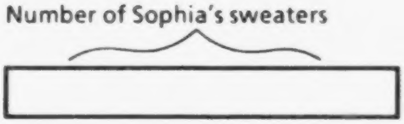
   If ten teams enter a tournament and each team plays every other team, how many games must be scheduled?

RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS[1]

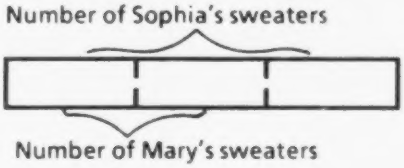
A. *Directions:* On this worksheet you will learn how to use diagrams to represent relationships between quantities in a given situation. The procedure uses a rectangular region to represent one amount; then that diagram is modified to represent another amount. Study the example given in problem 1, then complete the diagrams in problems 2 and 3.

1. Mary has 2/3 as many sweaters as Sophia.

   a. This region represents the number of sweaters that Sophia has.


Number of Sophia's sweaters

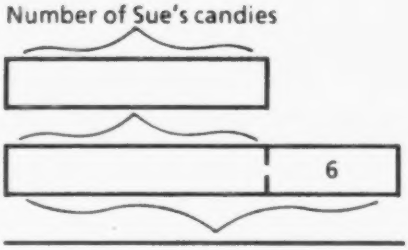
   b. The initial diagram is then modified to include the number of sweaters that Mary has.


Number of Sophia's sweaters
Number of Mary's sweaters

2. Roosevelt has 6 more candies than Sue.

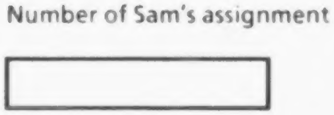
   a. Let the diagram represent the number of candies that Sue has.

   b. Write the missing label for the modified diagram.


Number of Sue's candies
6

3. Cynthia has 4 more than twice as many mathematics problems to solve tonight as her younger brother Sam.

   a. Let the diagram represent the number of problems in Sam's assignment.


Number of Sam's assignment

   b. Modify the diagram to show the relationship between the number of problems in the two assignments.

RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS (continued)

B. *Directions:* Draw and label diagrams to represent the relationship between quantities given in each of these problems.

1. Ronnie has 10 fewer baseball cards than Ritchie.

2. Elliot washed dishes 3 times as often as Bill.

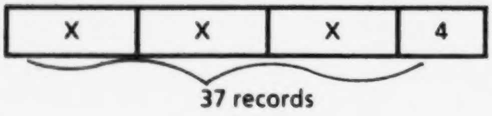
3. Manuel saw 4 more than 3 times as many baseball games as Luiz.

4. Vera practised 1 hour more than half as many hours as Tom.

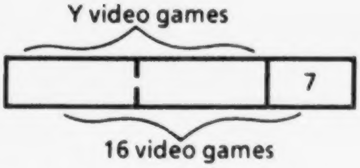
5. Deanna worked 5 less than twice as many days as Tisha.

RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS (continued)

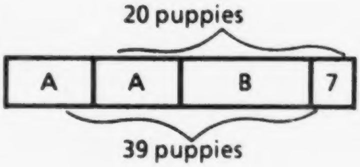
C. *Directions:* On this worksheet you are given diagrams that show relationships between certain numbers. The letters used in the diagrams represent unknown numbers. Dashed lines divide regions into congruent parts. Use the diagrams to find the values of the indicated quantities.

1. 
X
X
X
4
37 records

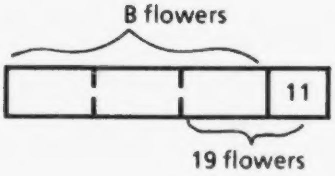
X = ______ records.

2. 
Y video games
7
16 video games

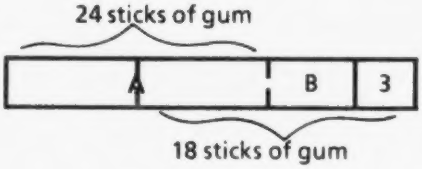
Y = ______ games.

3. 
20 puppies
A
A
B
7
39 puppies

A = ______ puppies.

4. 
B flowers
11
19 flowers

B = ______ flowers.

5. 
24 sticks of gum
A
B
3
18 sticks of gum

B = ______ sticks.

(*Hint*: First, figure out the number represented by half of A plus B.)

RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS (continued)

D. *Directions:* On this worksheet, diagrams are used to solve word problems. Draw and label a diagram for each problem, then use the diagram to answer the question.

1. Nancy and Barbara are collecting craft sticks to send in for a prize. When they first started collecting, they had 30 sticks between them. Barbara has not collected any more, but Nancy has doubled the number of sticks she has. They now have 42 sticks. How many sticks did Nancy have in the beginning?

   Diagram: Answer: ________________

2. My two dogs, Mutt and Jeff, were eating 34 cans of dog food each week. The veterinarian put Mutt on a diet. Now he eats only 1/2 as much as before. As a result, the dogs are eating only 24 cans each week. How many cans of dog food has Jeff been eating all along?

   Diagram: Answer: ________________

3. Jennifer and Angeli share a locker at school. They have 14 textbooks between them. When classes change at the semester break, Jennifer will have half as many books, but Angeli will have 4 more. They'll still have 14 textbooks to cram in their locker. How many do they each have now?

   Diagram: Answer: ________________

4. Sam and Janet were bundling newspapers for the school's paper drive. Between the two of them, they had 20 bundles at the start of the afternoon. Sam gathered 8 more bundles and Janet gathered the same amount she already had. This effort gave them a total of 35 bundles. How many bundles did Sam have at the beginning of the afternoon?

   Diagram: Answer: ________________

5. Ryan and Clark decided to raise rabbits because they wanted to have lots of animals to sell. They started with 18 rabbits, some white and some brown. After three months, they now have 3 times as many white rabbits and 2 times as many brown rabbits, for a total of 46 rabbits. How many white rabbits do they have after the three months?

   Diagram: Answer: ________________

---

[1] Reprinted with permission from *The Mathematics Teacher*, copyright 1989 by the National Council of Teachers of Mathematics.

RESOURCE 3: SPOT (SOCIAL PROBLEM-SOLVING STRATEGY)[1]

Teachers are encouraged to use the following social problem-solving strategy as a model when dealing with individual students or as a guide for students to use on their own.

Description of Strategy

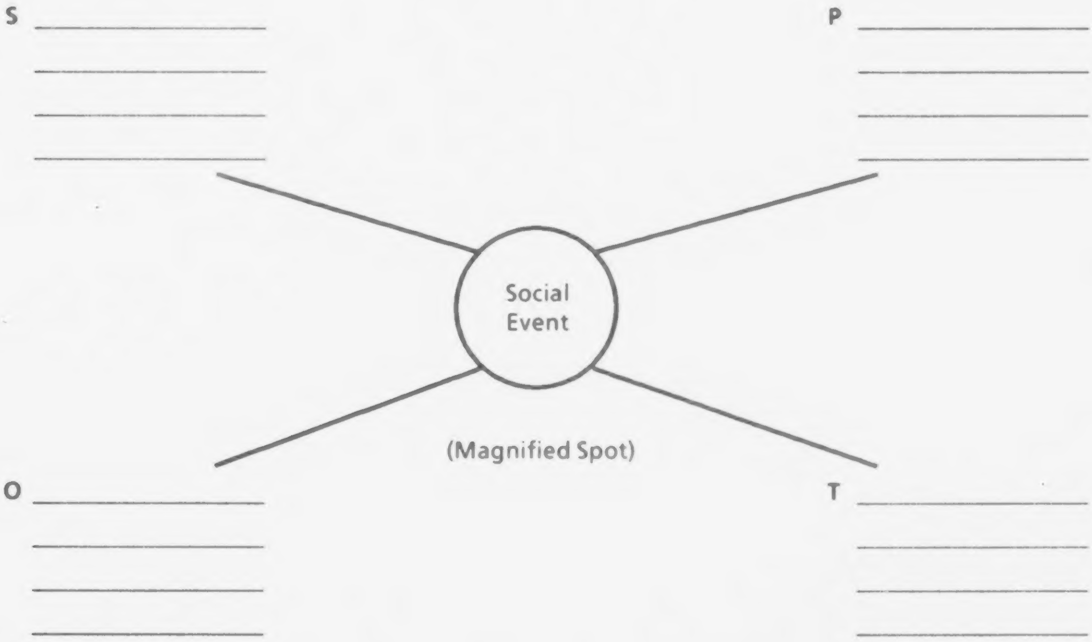
S: Setting: Who? What? Where? When?

P: Problem: What's the situation to be solved?

O: Order of action: What happened?

T: Tail End: What can be done next time?

Problem-Solving Chart:



(Magnified Spot)

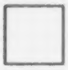
---

[1] From *Strategies for Teaching Students with Learning and Behaviour Problems* by Dr. C. Bos, and S. Baughn. Copyright 1988 by Boston, Allyn and Bacon. Reprinted by permission.

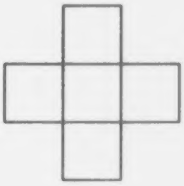
RESOURCE 4: PROBLEM-SOLVING IDEAS

1. LOOK FOR A PATTERN

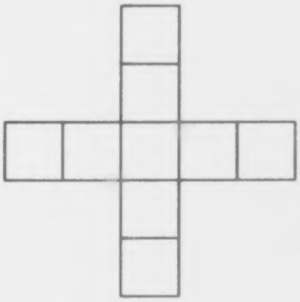
a. The 1st figure contains 1 square.
The 2nd figure contains 5 squares.
The 3rd figure contains 9 squares.



1st



2nd



3rd

If you made a drawing of the 4th figure, how many squares would it contain? How many squares would the 10th figure contain?

b. In each case, find the rule that will give the second number if you know the first. Then fill in the rest of the table according to the rule.

8	3
15	10
30	25
50	?
25	?
?	100

1	1
2	8
3	27
?	64
6	?
?	1000

2	7
3	10
5	16
8	?
22	?
?	22

c. Complete these patterns:

- 2, 5, 8, 11, ___, ___, ___, ___
- 1, 4, 9, 16, ___, ___, ___, ___
- 64, 32, 16, 8, ___, ___, ___, ___
- 1, 2, 4, 7, ___, ___, ___, ___

d. A school has 100 lockers and an enrolment of 100 students. All the doors are neatly closed for the summer holiday. On the first day of school, the first student to arrive opens all the locker doors. The second student to arrive closes every door with an even number. The third student reverses the position of all locker doors that have a number divisible by three (e.g., open becomes closed, closed becomes open). The fourth student reverses the position of all doors that have a number divisible by four. This pattern of events continues until all 100 students have entered the school.

Is locker number 100 opened or closed after the last student enters the school?

Teacher Note:
The number of lockers and students may be adjusted according to student ability. Start with 10 lockers and 10 students. Draw a diagram and use an "O" or "C" to indicate whether a particular locker is opened or closed.

Encourage students to use the number of factors of each locker number as a clue to the solution. Students should note that only numbers that are perfect squares will have an odd number of factors.

2. MAKE A SYSTEMATIC LIST

a. Two dice are thrown. How many different totals of the two dice are possible?

Complete this listing:

1st die	2nd die	Sum
1	1	2
1	2	3
1	3	4
●	●	●
●	●	●
●	●	●

b. The Canadian Football League has an interlocking schedule where each team plays every other team twice. How many games would have to be played? Make a season schedule for these games.

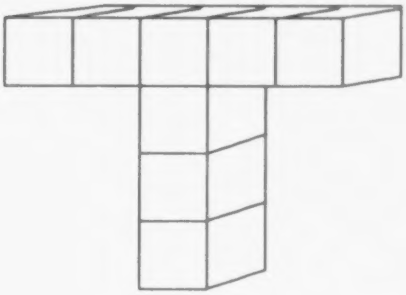
Western Conference	Eastern Conference
B.C. Lions	Winnipeg Blue Bombers
Edmonton Eskimos	Hamilton Tiger Cats
Calgary Stampeders	Toronto Argonauts
Saskatchewan Roughriders	Ottawa Roughriders

c. Kelvin has:
3 pairs of slacks – grey, black, blue
2 dress shirts – white, beige
3 sports jackets – plain, plaid, tweed.

How many different outfits can Kelvin make from this wardrobe?

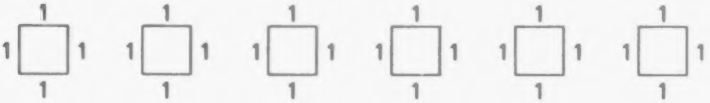
3. USE A MODEL OR DRAWING

a. Eight one-centimetre cubes are put together to form the T-shaped figure shown below. The outer faces of the T-shaped figure are painted red. The one-centimetre cubes are then separated. How many of the cubes have exactly four red faces?[1]

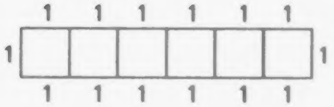


b. A restaurant has 6 square tables. Each table can accommodate one person on each side. How many different seating arrangements can be made using the 6 square tables?

Examples:



= 24 people



= 14 people

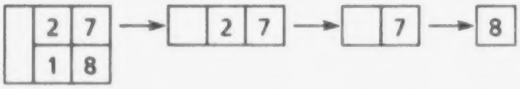
---

[1] Alberta Education. *Problem Solving Challenge for Mathematics*, 1985, p. 27.

c. Describe how to fold this "map" so that numbered sections lie on top of one another in order from 1 to 8.[1]

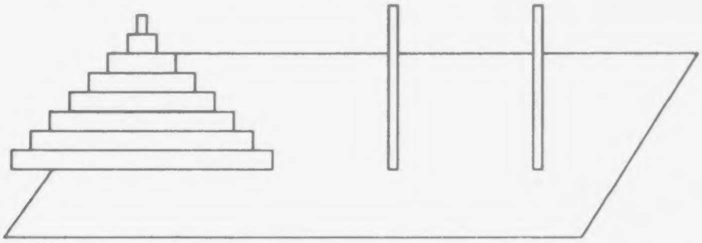
3	4	2	7
6	5	1	8

Solution:



d. TOWER OF HANOI

The "Tower of Hanoi" is a very old puzzle. The playing board consists of a flat surface with three vertical pegs. Seven rings with different radii are placed on one peg, in order of size with the largest ring on the bottom. The objective of the game is to transfer, using the least number of moves, all of the rings to another peg so they are arranged in the same order (i.e., with the largest ring on the bottom). At no time may a larger ring be placed on a smaller ring. The uppermost ring is the only ring in a pile that can be moved.



Teacher Note:
The puzzle can be easily made using paper tubes and cardboard rings. Adjust the difficulty of the game by increasing/decreasing the number of rings used. The least possible number of moves with seven rings is 127. If n rings are used, the least number of moves is $2^n - 1$.

---

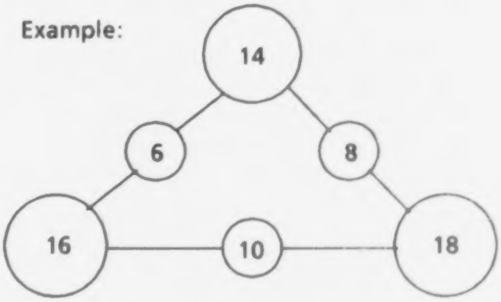
1. Alberta Education. *Problem Solving Challenge for Mathematics*, 1985, p. 47.

RESOURCE 4: PROBLEM-SOLVING IDEAS (continued)

4. GUESS AND CHECK

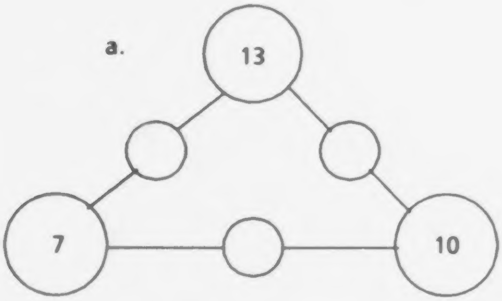
The numbers in the big circles are found by adding the numbers in the small circles.

Example:

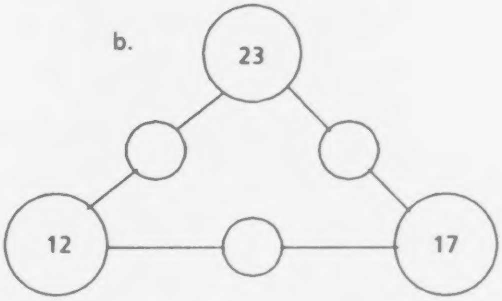


Find the numbers for the small circles in each problem.

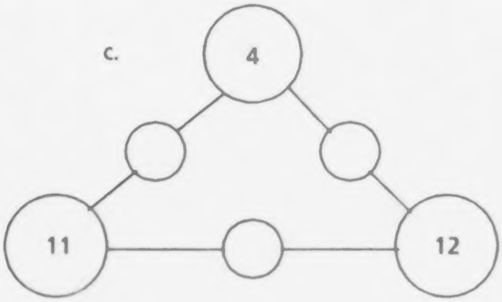
a.



b.



c.



5. A WINNING STRATEGY

Two players, A and B, have a pile of 6 toothpicks. Players alternately remove 1 or 2 toothpicks from the pile. The player who takes the last toothpick wins.

Teacher Note:
This game does not take long to play so students may be able to play it several times to discern the winning strategy. Students may discover several different strategies for winning. (e.g., The player who makes the second move always does the opposite to what the first player did. That is, if player A makes the first move and removes one matchstick, then player B will remove two, and so on.) The teacher may extend this game by changing the original number of toothpicks to 21, and instructing students to move one, two or three toothpicks at a time.

6. VISUALIZE THE PROBLEM

A tape is placed snugly around the earth at the equator. If another tape that is two metres longer is placed a uniform distance above the first tape, how far above the ground would it be? (Assume that the earth is a smooth sphere with a circumference of 40 000 km.)[1]

7. GUESSING AND CHECKING – ELIMINATING POSSIBILITIES

Choose a three- or four-digit number having no digits repeated. Ask someone to guess the number. Provide clues as to how close the guess is to the number chosen:

- say "Zilch" if no digits are correct
- say "Fermi" if there is a correct digit, but in the wrong position
- say "Pico" if there is a correct digit in the correct position.

For example, if the number chosen was 236, and someone guesses 532, the response is pico-fermi (i.e., one digit in the right position and another correct digit in the wrong position).

It may be advisable to choose two-digit numbers when introducing this activity. Students may play the game in pairs, keeping a record of the number of guesses made by each before determining the correct number. The winning student determines the number chosen with the least number of guesses.

8. LOGICAL THINKING

A used car dealer complained to his friend about having had a bad day. He told his friend that he sold two cars at $750 each. One of the sales yielded a profit of 25%. The other sale yielded a loss of 25%. "What are you worrying about?" asked his friend. "You had no loss whatsoever." "On the contrary, a substantial one", answered the car dealer. Who was right?[2]

9. IDENTIFY KEY WORDS

a. The average weight of four football players is 90 kg. The weights of three of the players are 82 kg, 92 kg, and 108 kg. What is the weight of the fourth player?[3]

b. Three friends ate lunch together. One person spent $3.60 for a glass of milk, two hamburgers and a donut. The second person bought two glasses of milk, one hamburger and two donuts for $3.00. If the third person bought one glass of milk, one hamburger and one donut, how much did this person pay?[4]

c. A freight train 500 m long passes through a 2000 m tunnel. If 60 seconds elapse from the time when the last car enters the tunnel to the time when the engine emerges from the other end, what is the speed of the train?[5]

---

[1] Alberta Education. *Problem Solving in Mathematics: Focus for the Future*, 1987, p. 67.
[2] Alberta Education. *Problem Solving Challenge for Mathematics*, 1985, p. 40.
[3] Ibid. p. 42.
[4] Ibid. p. 22.
[5] Ibid. p. 39.

10. ALGEBRA PROBLEMS

Arthur found this equation in his brother's algebra book:

$15N + 23 = 398$ (Remember, 15N means 15 times N.)

His brother explained that this equation is like a puzzle problem:
I'm thinking of a number. If you multiply it by 15 and add 23 you get 398. What is my number?

CLASS EXERCISES

(a) Arthur's first guess was 12. Was his guess too large or too small?
(b) Arthur's next guess was 30. Was the guess too large or too small?
(c) What is the solution to $15N + 23 = 398$?
(d) Here are other equations Arthur found in the book. First read the equation as a puzzle problem. Then find the solution.

$2N - 4 = 24$

$$\frac{4A+6}{2} = 7$$

$$\frac{M}{5} + 1 = 3$$

$$\frac{2N}{5} - 4 = 0$$

USE OF TECHNOLOGY

THE CALCULATOR

The calculator has become an integral part of our way of life. Students must become proficient and discerning in its use as an aid to computation and problem solving. **All students shall have calculators readily available for use throughout the program.**

Effective use of the calculator requires an understanding of place value and the ability to judge the reasonableness of the results of calculations. Skills in estimation and mental arithmetic become increasingly important in enabling students to anticipate and verify calculator results. These experiences will be more effective in developing number sense and cognitive process than long and tedious computations with paper and pencil. Competence in estimation and mental arithmetic can be expected to improve through frequent use of these skills while using a calculator. By using calculators appropriately, time previously spent on tedious computation can be used to develop cognitive process and problem-solving skills.

The guidelines that follow are intended to provide a classroom structure that facilitates appropriate use of the calculator as an instructional and computational tool.

- The calculator does not reduce the need for basic arithmetic skills. **Effective use of the calculator requires that an understanding of place value, basic number skills, and arithmetical operations is emphasized throughout the instructional program.**
- There will always be a need to possess limited paper-and-pencil computational skills. **For most students, the calculator should be used after the mathematical concepts and algorithms are understood.** Students who simply cannot master their basic number skills or who have persistent problems with computation should use calculators more extensively.
- The calculator should always be used in situations where long and/or extended computations are required.
- **When using the calculator in problem situations, place emphasis on estimation and mental arithmetic**, on proper documentation of numbers and operations used, and on the reasonableness of answers.
- Do not assume that students understand how to use a calculator. **Be prepared to teach students how and when to use a calculator properly.** Remember, however, that creating a situation just for the sake of using calculators will not develop an understanding of their appropriate use.

A variety of suggestions and ideas for developing effective calculator skills have been included in this section of the manual, and focus attention on:

- Entry Procedures
- Order of Operations
- Auditory Cueing
- Rounding Results
- Finding Remainders
- Using the Percent Key
- Using the Memory Keys
- Checking the Reasonableness of Results
- Developing Confidence and Speed.

ENTRY PROCEDURES

Provide opportunities for the development of basic entry skills on the calculator. Most errors made by students in using their calculators are due to incorrect entries that have been made. Students often enter numbers for division/subtraction/exponentiation in the wrong sequence, and frequently experience difficulty with questions that involve multiple entries.

Ensure that students are familiar with the basic operations of their calculators. Keys operated include:

+	addition	X	multiplication	$\sqrt{X}$	square root
-	subtraction	C	clear		
÷	division	=	equals	y^x	exponentiation.

Use an overhead transparency to demonstrate appropriate procedures for entering numbers and performing operations on the calculator. Assist students to develop confidence in using these procedures by providing both oral and written exercises.

CLARIFICATION/EXAMPLE

Procedure for Decimal Operation
(e.g., Find the mean of 8.1, 9.2, 7.6.)

	Entry	Display
Turn on		0.
Enter	8 . 1	8.1
Enter	+	8.1
Enter	9 . 2	9.2
Enter	+	17.3
Enter	7 . 6	7.6
Enter	+	24.9
Enter	÷ 3	3.
Enter	=	8.3

CLARIFICATION/EXAMPLE

Procedure for Calculating Exponent Values and Square Roots:
(e.g., 4^3)

	Entry	Display
Enter	4	4.
Press	y^x	4.
Enter	3	3.
Press	=	64.

(e.g., $\sqrt{6.25}$)

	Entry	Display
Enter	6 . 2 5	6.25
Enter	$\sqrt{\ }$	2.5

ORDER OF OPERATIONS

Students need to perform operations in correct sequence when working with multiple step calculations and problems. Illustrate the order of operations on a poster in the classroom, and provide students with a mnemonic device that will facilitate recall of the skills they develop. Remind students that exponentiation is a form of multiplication and should be performed prior to any addition or subtraction.

CLARIFICATION/EXAMPLE

The "BODMAS" Mnemonic For Order of Operations

B	O	D	M	A	S
↓	↓	↓	↓	↓	↓
Brackets	Of	Divide	Multiply	Add	Subtract

The problems that follow illustrate the importance of performing operations in the correct sequence. Although each problem may be solved through one set of key strokes on some calculators, students should be encouraged to document the process used and record intermediate solutions.

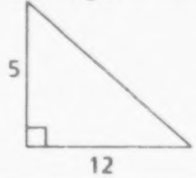
CLARIFICATION/EXAMPLE

1. Jane used her calculator to compute the number of calories in the snack she ate after school. She had a coke with 145 calories, 18 pretzel sticks with 4 calories each, and 7 potato chips with 12 calories each. She entered the following numbers on her calculator:

   $145 + 18 \times 4 + 7 \times 12 =$

   To her horror, the calculator showed 7908 calories! Jane knew that her allowance of calories for an entire day was only 2280 calories. What went wrong?

2. Calculate the length of the third side.



$$C^2 = 5^2 + 12^2$$
$$C = \sqrt{5^2 + 12^2}$$

Activities that may assist students to perform sequential operations on their calculators are provided in 'Resource 1: Using Brackets and Parentheses".

AUDITORY CUEING

The wording of directions and problems that involve arithmetical operations is often a source of confusion for students. Variations in wording may cause students to enter numbers into the calculator in the wrong sequence. Be consistent with the wording/auditory cues you provide for students. Students should develop the ability to handle alternative auditory cues, but only after basic skills and procedures are understood.

CLARIFICATION/EXAMPLE

Alternative Auditory Cues for Subtraction

- subtract 3 from 6
- 6 minus 3
- take away 3 from 6
- 6 take away 3
- What is the difference between 6 and 3, or 3 and 6?

Alternative Auditory Cues for Division

- divide 6 by 3
- 6 divided by 3
- 3 divided by 6
- What is 6 over 3?

Alternative Auditory Cues for Exponentiation

- 4 squared
- 4 to the power of 2
- 4 to the second power
- the second power of 4

ROUNDING RESULTS

Students need to learn how to interpret results of the computations they perform on the calculator. This is particularly true in division, where digits are displayed to the right of the decimal point when the answer is not a whole number. Encourage students to consider the context in which the computation was performed, and to determine appropriate procedures for rounding. (e.g., Should the answer be rounded to the correct tenth or hundredth?)

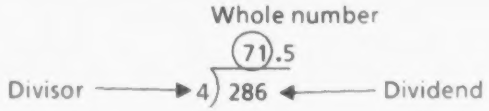
A strategy that may be effective in helping students to understand the process of rounding numbers is provided in Computational Facility and Estimation, "Developing Strategies for Estimation".

FINDING REMAINDERS

Division may be a difficult operation when performed by hand or on the calculator. If the divisor is a factor of the dividend, the process is relatively simple on the calculator. If the divisor is not a factor of the dividend, however, students must recognize that the answer can be expressed as a decimal (which may require rounding) or as a whole number with remainder. As some problem situations require exact remainders to be known, students should be familiar with procedures for finding whole number remainders using numbers displayed on the calculator.

CLARIFICATION/EXAMPLE

What is the whole number remainder for 286 ÷ 4?



Step 1: Multiply the divisor by the whole number in the answer.

$4 \times 71 = 284$

Step 2: Subtract the result of step 1 from the dividend to obtain the remainder.

$286 - 284 = 2$ (remainder)

Students may benefit by initially documenting the process they use for finding whole number remainders in a table or chart. The table provides a structure for the process, and enables students to record intermediate steps and solutions in an effective way.

CLARIFICATION/EXAMPLE

Question	Calculate Answer	Step 1 Multiply Divisor by Whole Number	Step 2 Subtract Answer in Step 1 from Dividend	Remainder
$8\overline{)47}$				
$4\overline{)550}$				
$2\overline{)961}$				
$62\overline{)4217}$				

USING THE PERCENT KEY

There are two main types of calculators, each using a different logic system as its base. One is "algebra-based", and the other is "arithmetic-based". The calculators do not differ in appearance, but in how calculations are executed. While this duality of logic systems does not present a problem in most instances, it does affect how the percent function operates. Students should recognize that:

- if the calculator is algebra-based, pressing the percent key causes the number in the display to be divided by 100
- if the calculator is arithmetic-based, the percent key can only be used as a multiplier.

Ask students to determine which logic system their calculator uses by entering the number 25 and then pressing the percent key. If the display reads 0.25, the calculator is algebra-based. If the display does not change, reads 0 or E, the calculator is arithmetic-based.

CLARIFICATION/EXAMPLE

Find 25% of 36.

Key strokes on algebra-based calculator:

$\boxed{25}\ \boxed{\%}\ \boxed{\times}\ \boxed{36}$ or $\boxed{25}\ \boxed{\times}\ \boxed{36}\ \boxed{\%}$

Key strokes on arithmetic-based calculator:

$\boxed{36}\ \boxed{\times}\ \boxed{25}\ \boxed{\%}$

USING THE MEMORY KEYS

Most hand-held calculators have a memory storage system. Although the steps to follow in using memory will vary according to the type of calculator being used, most memory systems are quite similar. Memory systems usually consist of four keys:

M+ M− CM RM

The M+ key is used to place the contents of the display into memory, or to add the contents of the memory to the display. The M− key is used to subtract the contents of the display from memory, or to subtract the contents of memory from the display. The CM key (or MC key on some calculators) is used to clear the memory. Remember always to clear the memory before starting a new set of calculations. The RM key (or MR key on some calculators) denotes memory recall and will display the contents of the memory.

The following examples illustrate use of the four memory keys.

CLARIFICATION/EXAMPLE

$(12\times 10) + (2\times 8) =$

Entry	Display
CM	0.
12	12.
×	12.
10	10.
=	120.
M^+	120.
2	2.
×	2.
8	8.
=	16.
M^+	16.
RM	136.

CLARIFICATION/EXAMPLE

$(12\times 10) - (2\times 8) =$

Entry	Display
CM	0.
12	12.
×	12.
10	10.
=	120.
M^+	120.
2	2.
×	2.
8	8.
=	16.
M^-	16.
RM	104.

Model appropriate strategies for using the memory key when solving problems that involve several steps and operations. Frequent practice will enable students to develop confidence in using the calculator's memory.

CHECKING THE REASONABLENESS OF RESULTS

Encourage students to anticipate and verify the results they obtain on their calculators, and to monitor their own work through sub-vocal rehearsal and questioning.

e.g.,
- Is my answer reasonable?
- Within what range of numbers must my answer lie?

Model strategies that will enable students to estimate the results of computations they are about to perform on the calculator. A variety of effective estimation strategies are provided in the "Computational Facility and Estimation" section of this document.

CLARIFICATION/EXAMPLE

Question	Estimated Answer	Calculated Answer
$48 + 50 =$		
$\$10.00 - \$1.96 =$		
$7^3 =$		
$37 \div 7 =$		
$26 \times 9 =$		
$\sqrt{30} =$		
$47 + 80 =$		
$\$3.01 \div \$0.98 =$		

CALCULATOR GAME OF 'ESTIMATION' (2 players)

Two players each have 30 seconds to estimate a product or quotient of two-, three- or four-digit numbers. The player with the closer estimate wins a point. The first player with ten points wins.

DEVELOPING CONFIDENCE AND SPEED

Confidence and skill in performing computational procedures on the calculator will increase as students gain experience in using the calculator in a variety of contexts. Provide opportunities for students to develop and refine calculator skills in situations that involve:

- drill and practice
- timed challenges
- problem solving
- puzzles and games.

Sample puzzles and games that may be useful in developing calculator skills are provided in "Resource 2: Calculator Puzzles and Games".

CLARIFICATION/EXAMPLE

CALCULATOR GAME OF '21' (2 players)

On each turn a player can push only one of the three keys $\boxed{1}$, $\boxed{2}$ or $\boxed{3}$ along with the $\boxed{+}$ key. The first player to reach 21 is the winner. Ask students to determine the winning strategy.

CALCULATOR GAME OF '50' (2 players)

This game is similar to the game of '21'. Players may use the $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$ and $\boxed{6}$ keys. The first player to reach 50 is the winner.

THE COMPUTER

Current demands of society place emphasis not only on the ability to communicate through the written word, but also on the skills of interacting with computer technology. The mathematics program must provide opportunities for students to demonstrate an understanding of:

- the basic operation of computers, by
  - identifying major parts of a computer
  - distinguishing between hardware and software
  - recognizing that computers get their instructions from a program written to accomplish a specific task.

- the basic capabilities, applications and limitations of computers, by
  - recognizing that computers are best suited to tasks requiring speed, accuracy, repeated operations and the processing of large amounts of data
  - identifying major areas in society where computers are used, and the tasks performed by computers in these areas
  - recognizing tasks that computers cannot accomplish.

To the extent that facilities and equipment are available, the mathematics program should also provide opportunities for students to:

- work independently with prepared software
- enter, run and modify simple programs that have been written for particular purposes.

This section of the manual describes two "user friendly" programming languages that will enable both student and teacher to interact with the computer without relying solely upon commercially prepared software. Commands and procedures provided on the pages that follow will enable students to enter, run and/or modify simple programs in:

- BASIC computer language
- LOGO computer language.

BASIC computer language has been selected because most microcomputers use this language. The use of BASIC may vary slightly in response to the type of computer being used. The programs described throughout this manual refer to the BASIC programming language used on an Apple computer.

LOGO computer language has been selected because of its simplicity and graphic capabilities. Different versions of LOGO are also available. The version described in this manual is Terrapin LOGO.

Although neither BASIC or LOGO require extensive programming knowledge, the degree to which they are used within the mathematics program will depend upon student interests/abilities and available resources. Teachers are encouraged to adapt and modify simple programs as required in meeting the needs of individual students and/or classes.

BASIC COMPUTER LANGUAGE

Most microcomputers are capable of interpreting BASIC language commands. A disc is not needed to boot the computer for BASIC as the computer will automatically interpret the commands. A disc is only needed if the program is to be saved. When using the computer without a disc in the drive, the drive will "whir" for a time before the cursor appears. To stop the whirring on your own, press "control reset". The cursor should appear in the lower left hand corner.

A list of common commands used in BASIC programming is provided below.

Key Word	Description
DATA	DATA 45, 67, 35 Holds the data for use by READ statement.
END	Terminates the program.
FOR	FOR X = 1 to 10 Substitutes the numbers from 1 to 10 into the program. Causes a LOOP to occur.
GOTO	GOTO 50 Tells the computer to go to line 50 of the program.
HOME	Clears the screen and sends the cursor to the upper left hand side of the screen.
IF . . . THEN	IF X > 1 THEN 90 The computer makes a decision as to the truth of a statement and if true, in this example, will proceed to line 90. If the condition is not true it will go to the next line.
INPUT	INPUT :N$ Calls for input from the keyboard during program execution.
INT	PRINT INT (23.6) Will give the largest integer less than or equal to the value given. In this case the computer would print 23.
LET	LET X = 10 Assigns a value to a variable.
NEXT	NEXT X Closes a FOR . . . NEXT loop.
PRINT	PRINT "HELLO" The computer will print the expression in the quotation marks.

Key Word	Description
READ	READ X, Y, Z Reads values from a DATA statement.
REM	REM - - "THIS IS A COMPUTER PROGRAM" The REM command allows the programmer to insert remarks for explanation into the program.
STEP	FOR X = 0 to 100 STEP 10 The computer will substitute the numbers from 0 to 100, counting in increments of 10.

The arithmetic commands in BASIC include:

+ – addition
- – subtraction
* – multiplication
/ – division
^ – exponentiation

Additional commands that may be needed to program in BASIC include:

Command	Execution
LIST	Displays the entire program contained in the computer's memory. If the program is long it may be necessary to display only part of it at a time. This can be accomplished by listing the lines to be displayed. For example, LIST 10 – 100 would display all lines from 10 to 100.
LOAD	To retrieve a program from a disc, type LOAD and the name of the program.
NEW	Before entering a program, type NEW to clear the computer memory.
RUN	After a program has been entered type RUN to have the computer execute the program.
SAVE	If the program is to be saved on a disc, type SAVE and the program name.

When programming in BASIC, one must realize that the computer executes commands in order of their line number. It is important that all lines be numbered, and that they are numbered sequentially according to the order of the commands.

CLARIFICATION/EXAMPLE

A BASIC program for finding the factors of a number:

```
 10  REM FIND ALL FACTORS OF A NUMBER
 20  PRINT "WHAT IS THE NUMBER"
 30  INPUT N
 40  PRINT "THE FACTORS ARE: "
 50  FOR K = 1 TO N
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END
```

Notice that the line numbers are listed in increments of ten. This allows the programmer to insert a new line without having to re-type the rest of the program. For example, if the programmer wished to insert a new line between 10 and 20 it could be entered as line 15 and the computer will automatically execute the new line in its numerical order.

Here is an explanation of the program.

Line Number	Execution
10	The REM command has allowed the programmer to state the purpose of the program. Note that the REM statement has no effect on the running of the program. It is only there for explanatory purposes.
20	When the program is run the statement "WHAT IS THE NUMBER" will be printed.
30	INPUT N calls for the operator to input the number to be factored.

CLARIFICATION/EXAMPLE

Line Number (continued)	Execution (continued)
40	After the operator inputs the number, the computer will print the statement: "THE FACTORS ARE". Note that the statement to be printed is enclosed in quotation marks.
50	FOR K = 1 to N tells the computer to test the numbers from 1 to N (the number specified in INPUT) in increments of one. Notice that the STEP statement does not need to be included if the step is one.
60	The LET statement defines the variable X as the quotient of N ÷ K.
70	The LET statement here defines the variable Y as the integer value of N ÷ K. The integer value is the greatest integer not larger than N ÷ K.
80	The conditional statement IF . . . THEN tests a condition and begins a loop. In this case the computer is determining if X = Y. That is, when N is divided by K, does it result in an integer (no remainder)? If the statement is true then K will be printed.
90	NEXT K tells the computer to test the next value of K. Remember, the computer will start testing at 1 and end at N.
100	When the computer reaches this line number it will cease executing the program.

Run the previous program to be sure that it works. A sample run would be:

Type RUN

The screen should read:

```
WHAT IS THE NUMBER
```

Enter a number (e.g., 36). The computer screen will now look like this:

```
THE FACTORS ARE:
  1
  2
  3
  4
  6
  9
 12
 18
 36
```

Once a program has been run, the teacher may wish to make revisions to the program. Revisions may often involve adding additional lines and commands to the original program.

CLARIFICATION/EXAMPLE

A revised BASIC program for finding the factors of a number:

```
 10  REM FIND ALL FACTORS OF A NUMBER
 20  PRINT "WHAT IS THE NUMBER"
 30  INPUT N
 40  PRINT "THE FACTORS ARE: "
 50  FOR K = 1 TO N
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END

 25  PRINT "YOU WISH TO FACTOR"
 95  PRINT "DO YOU HAVE ANOTHER NUMBER"
 96  INPUT R$
 97  IF R$ = "YES" THEN 20
 98  PRINT "OK! GOOD BYE FOR NOW."
```

Line 25 will be inserted between lines 20 and 30. It will cause the computer to print YOU WISH TO FACTOR. This insertion only serves to clarify the statement WHAT IS THE NUMBER. The entire statement could not be placed on line 20 because there are only 27 character spaces left on the line after the print statement.

Line 95 is the first of three lines that allow the program to be repeated. When the program is run after the initial list of factors, the computer will ask if another number is wanted.

Line 96 calls for input from the operator. Note the form of the input, R$. This is called a string variable. It prepares the computer for input which may be alphabetic or numeric, and which may be several characters in length. If the input is to be a word or letter, a string variable must be used.

Line 97 is the conditional statement. If the input from the operator is YES (not just Y), the computer will go back to line 20 and begin the factoring process again. If the input is anything other than YES the computer will go on to the next line.

Line 98 causes the computer to print "OK! GOOD BYE FOR NOW.".

The examples and explanations provided on BASIC language are fairly simple and include only those commands that a novice may use. Teachers may wish to consult other programming texts for additional commands and strategies that are used when programming in BASIC.

LOGO COMPUTER LANGUAGE

The LOGO computer language is an interpretive language in that commands may be executed immediately, or can be stored for use in "procedures". A procedure is a group of one or more instructions and commands that the computer stores for future use. Once a procedure has been defined, it may be used as a part of other procedures.

There are many facets to the LOGO language, but perhaps the most appealing characteristic of LOGO is its graphic capability. Very little knowledge of the language is required prior to using the graphics mode of LOGO. This section will highlight some of the graphics commands to be used in Terrapin LOGO. Be prepared to make adjustments to these commands if using another version of LOGO (e.g., LOGO writer).

To enter the graphics mode, type DRAW. The LOGO turtle, Δ, should now appear in the middle of the screen. This is the turtle's HOME position. Once you are in the draw mode you can command the turtle to perform various moves. Some of the more commonly used graphics commands are provided below.

Graphics Command	Execution
HOME	This command places the turtle in his home position in the centre of the screen.
FD ______*	This command followed by a number will move the turtle forward that number of turtle steps. For example, FD 100 would move the turtle forward 100 steps.
BK ______*	This command followed by a number will move the turtle backwards by that number of turtle steps.
LT ______*	This command followed by a number will designate a left turn by that number of degrees. For example, LT 45 will turn the turtle to the left 45 degrees.
RT ______*	This command will turn the turtle by the desired number of degrees to the right.
PU	Means "pen-up", and ensures that the turtle's path that is executed after the PU command will not show.
PD	Means "pen-down". If a PU command has been used, it is necessary to type PD prior to any further commands to make the turtle's path visible.
CS	Means clear screen. This command erases all previous paths from the screen.

* Do not type a blank. The blank lines represent a need for numerical input.

To access the text mode of LOGO for programming, simply type TO and the name of your procedure. LOGO will automatically switch into the text mode.

The edit commands listed below will be useful in defining procedures and writing simple programs.

Edit Command	Result
CONTROL D (CTRL D)	Deletes character under the cursor.
CTRL P	Moves the cursor up one line.
CTRL N	Moves the cursor down one line.
CTRL E	Moves cursor to the end of the line.
CTRL G	Stops the procedure .
CTRL C	Defines the procedure.

CLARIFICATION/EXAMPLE

A sample procedure to be executed in LOGO:

```
TO   SQUARE  :S
FD   :S
RT   90
FD   :S
RT   90
FD   :S
RT   90
FD   :S
RT   90
END
```

Explanation:

```
TO SQUARE  :S
```

The TO tells the computer that you are going to define a procedure which you are naming SQUARE. Procedure names can only be one word. If it is necessary to have more than a one word description, the words may be separated with periods (e.g., TO SQUARE.MAZE). The :S indicates to the computer that there will be a variable in the procedure. In this case the variable is the length of the side.

The directions that follow instruct the turtle to move forward a specified number of steps, followed each time by a 90° right turn. Note that the final RT 90 is not necessary for drawing the square, but it is good programming practice to have the turtle end in the home position facing forward.

To run the procedure simply type SQUARE and a length of a side (e.g., SQUARE 10). The computer should now draw a square whose sides are 10 turtle steps in length.

The procedure for square could also be written as:

```
TO SQUARE :S
REPEAT 4 [FD :S  RT 90]
END
```

Defining the procedure this way makes use of the REPEAT command and shortens the program.

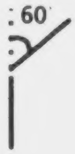
Once a procedure has been defined in LOGO, it may be used in other procedures.

CLARIFICATION/EXAMPLE

```
TO STACK.SQUARE :N :S
REPEAT :N [SQUARE :S FD :S]
END
```

If STACK.SQUARE 5 20 is entered, the computer will draw five squares whose side length is 20 turtle steps.

When drawing polygons with LOGO, encourage students to use the "total trip theorem". This theorem states that the turtle must turn through a total of 360 degrees in order to produce a closed figure. Instructions that cause the turtle to turn by a desired number of degrees (i.e., draw an angle) must therefore be stated in terms of the supplement of each angle in the polygon. The command "FD 10 RT 60 FD 10" would cause the turtle to draw a 120 degree angle in the manner illustrated below:



The "total trip theorem" will enable students to construct both acute-angled and obtuse-angled polygons. Right-angled polygons are more easily constructed as the command "RT 90" will produce an angle of 90 degrees (i.e., the supplement of 90 degrees is 90 degrees).

CLARIFICATION/EXAMPLES

A LOGO Program for Producing a Regular Pentagon

A pentagon has five angles and the turtle must turn through 360°. Therefore, each turn must be $\frac{360^\circ}{5} = 72^\circ$.

```
TO PENTAGON :S
REPEAT 5[FD :S RT 72]
END
```

After entering a procedure be sure to press CTRL C to have the computer put the definition into its memory. If you wish to save a procedure on a disc, type SAVE "FILE NAME". To read from the disc, type READ"FILE NAME".

This brief description of LOGO is intended to enable teacher and student to begin using the graphics and text modes of LOGO. Only the graphic capabilities of LOGO have been discussed. The arithmetic capabilities of the language could be investigated through the use of a LOGO manual.

RESOURCE 1: USING BRACKETS AND PARENTHESES

A. Calculate 165 + (168 × 337)

First multiply: (168 × 337) = ____________

Then add: 165 + 56 616 = ____________

Here is the sentence to use. 168 × 337 + 165 = ____________

B. Rewrite the following so you can use your calculator. Then do the question.

	Calculator Sentence	Answer
1. 2411 + (3768 × 72)	____________	____________
2. 180 164 × (165 + 28)	____________	____________
3. 273 × (8927 − 7373)	____________	____________

If there is more than one set of parentheses, sometimes square brackets are used.

[41 + (43 × 44)] ÷ 40
↑ ↑
Next Do first

Then do the rest of the work.

C. Complete the following:

	Calculator Sentence	Answer
1. [36 × (71 + 32)] − 5 =	____________	____________
2. [57 + (472 − 150)] ÷ 8 =	____________	____________
3. 586 + [35 + (483 ÷ 7)] =	____________	____________

RESOURCE 2: CALCULATOR GAMES AND PUZZLES

A. Your calculator can spell for you. Certain numbers on your calculator, when turned upside down, look like letters.

1. Find out which numbers produce the following letters:

O __ I __ E __ S __ L __ B __ G __ Z __

2. What word does each of the following numbers spell?

77 345 ______ 0.7734 ______

3504 ______ 3080 ______

3. Solve each of the following to find the word:

- a telephone greeting: (1 – .30394) ÷ 9 = _____ (hello)
- what a snake does: 471 × 265 + 410 699 = _____ (hisses)
- what water does at 100°C: 305 644 ÷ 43 = _____ (boil)
- where a turtle lives: 11 345 = _____ (shell)
- the name of a book: 4202 × .5 × 18 = _____ (bible)
- how math is for you: 16 × 3 × 5 × 83 × 277 = ______ (oh bliss)

4. Solve these expressions in order to answer the questions:

- What did Amelia Earhart's father say the first time he saw her fly an airplane?

  (.023 × 3 + 10 141) × 5 = _____ (she solos)

- How do fish breathe?

  5787 ÷ 3 × 4 = _____ (gill)

- What is a pit?

  59 × 59 + 223 = _____ (hole)

5. Create other words and phrases that are the results of pre-planned calculations.

RESOURCE 2: CALCULATOR GAMES AND PUZZLES (continued)

B. Guess the Number.

1. Ask a friend to enter any number (not more than six digits) on the calculator and then multiply by 2; add 4; multiply by 5; add 12; multiply by 10; press the "=" key.

2. Take the calculator back from your friend and subtract 320. The result will end in one or more zeros. Drop the zeros, and you'll have the number your friend started with.

Example: Choose 65219

- $65\,219 \times 2 + 4 \times 5 + 12 \times 10 = 6\,522\,220$.
- $6\,522\,220 - 320 = 6\,521\,900$; discard the two trailing zeros, and you have 65 219, the number your friend started with.

C. Sum of 1089.

1. Ask a friend to select any 3-digit number. (The first and third digits cannot be the same.)

2. Ask your friend to:
   - reverse the digits
   - find the difference between the two numbers (larger minus smaller).

3. Reverse the digits of the difference. Then find the sum of these two numbers. The answer will always be 1089.

   Example: Choose 155

   - Reverse the digits : 551
   - Find the difference : $551 - 155 = 396$
   - Reverse the digits : 693
   - Find the sum : $396 + 693 = 1089$

D. Your Birthday and Age.

1. Hand someone your calculator and ask them to enter the number representing the month of their birthday; multiply it by 100; add their date of birth; multiply by 2; add 9; multiply by 5; add 8; multiply by 10; subtract 422; add their age; then press the "=" key.

2. Take the calculator back and subtract 108. The result will be a five- or six-digit number. The first digit(s) tells the month of birth; the next two the date of birth; and the last two, the age of your friend. Your friend will be impressed.

Example: Your friend is 17 years old and was born on March 13.

- $3 \times 100 + 13 \times 2 + 9 \times 5 + 8 \times 10 - 422 + 17 = 31\,425$.
- Take $31\,425 - 108 = 31\,317$ or 3/13/15.

Your friend was born in the third month (March), on the 13th day, and is 17 years old.

COMPUTATIONAL FACILITY AND ESTIMATION

The availability and use of technology has caused emphasis in the skills required for computational competence to change over the last decade. Computational facility includes more than the knowledge and skills required to perform paper-and-pencil computations with standard algorithms. While these skills are important, responsible participation at home and at work also requires facility in performing mental arithmetic, in the application of calculator skills, and in applying strategies of estimation. Surveys show that mental computation and estimation are used in more than 80 percent of all practical problem–solving situations encountered outside the classroom.

Teaching and learning must focus on the development of a variety of computational strategies that include:

- paper-and-pencil process. Paper-and-pencil computation should focus on an understanding of process, and de-emphasize calculation with large numbers. Addition and subtraction should generally include numbers with no more than four digits. Multiplication and division should be performed on numbers containing up to four digits, using multipliers/divisors of no more than two digits
- the use of mental arithmetic. Students should have knowledge of basic number skills, and be able to apply these skills to more sophisticated processes. Short drill and practice activities that become part of the daily routine will promote the development of mental arithmetic skills and foster a habit for their use
- the use of a hand-held calculator. Calculator activities should emphasize an understanding of place value and the ability to judge the reasonableness of the results of calculations. Skill in estimation and mental arithmetic are important in enabling the student to anticipate and verify calculator results
- the use of estimation. Students need to be able to determine if a particular result is precise enough for the purpose at hand, and be alert to the reasonableness of computational results. In order to carry out rapid estimations, students must understand place value, have skill in single-digit operations, be able to multiply and divide by powers of ten, and have facility in rounding whole numbers and decimals to the number of significant digits required by the situation.

Suggestions and ideas for developing student ability to compute by a variety of methods have been included in this section of the manual, and focus attention on:

- Developing Computational Process
    - Understanding Place Value
    - Using Manipulatives
- Developing Mental Arithmetic Skills
    - Applying Basic Number Skills
    - Using Formal Strategies
- Developing Strategies for Estimation
    - Front-End Estimation
    - Rounding
    - Compatible Numbers
    - Clustering (or Averaging).

Suggestions for developing calculator skills are included in another section of this manual (see "Use of Technology").

210

DEVELOPING COMPUTATIONAL PROCESS

Instruction in paper-and-pencil computation should emphasize place value and basic number skills prior to the application of algorithms. Concrete and visual material will assist in concept development. Students who have not experienced past success with standard algorithmic procedures may react more favourably and experience more success with less sophisticated forms of algorithms.

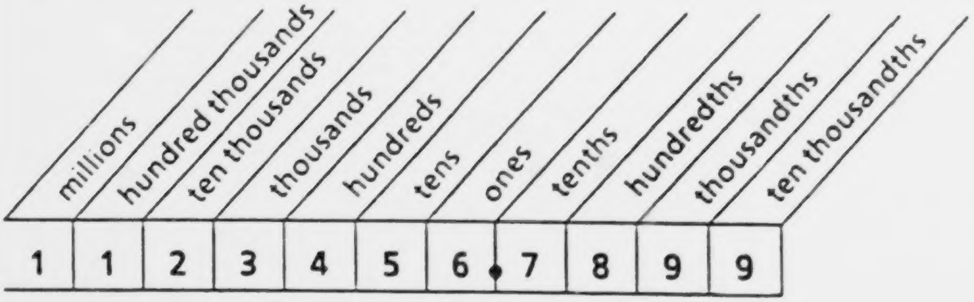
UNDERSTANDING PLACE VALUE

An understanding of place value is critical to the development of computational process. Develop and reinforce whole number and decimal place value through discussion, questioning and a variety of concrete/visual experiences. Teachers may find it beneficial to:

- display a large place value chart in the classroom that students can refer to as necessary

CLARIFICATION /EXAMPLE

Place Value Chart



- design activities that require students to apply their knowledge of place value to computations performed on the calculator.

CLARIFICATION/EXAMPLE

- Enter the number "21.455" into a calculator.
- Suggest an amount that can be added/subtracted to this number in order to obtain the following results:

  26.455 (+5)
  21.405 (−0.05)
  61.455 (+40)
  21.055 (−0.4).

- Verify each idea by performing the necessary operation on the calculator.

USING MANIPULATIVES

Manipulative materials provide students with a concrete base upon which to build concepts and skills. Students who experience difficulty with arithmetical process should be given opportunities to develop an understanding of number operations and relationships through tactile and visual learning activities.

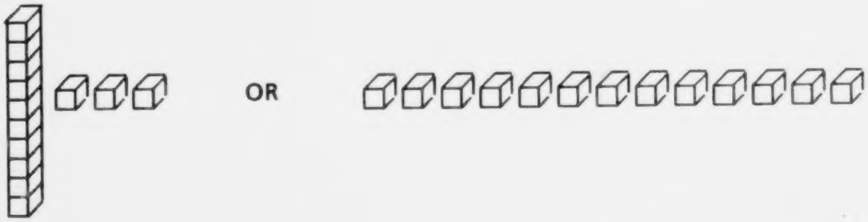
Manipulatives are particularly useful in clarifying the division process. Encourage students to recognize division as a "sharing process", and guide students in the use of appropriate thought processes by asking relevant chains of questions.

e.g.,
- How many are to be shared?
- How many are doing the sharing?
- Can we share so that everyone gets a ten? a hundred?
- What is the greatest number each will have when the sharing is done?
- Are there any left over?
- How can we determine the greatest number that can be shared?

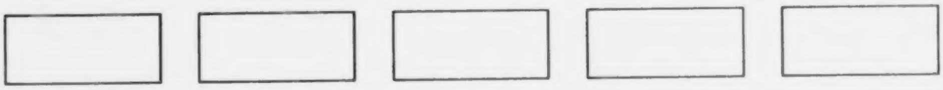
CLARIFICATION/EXAMPLE

13 ÷ 5 = ?

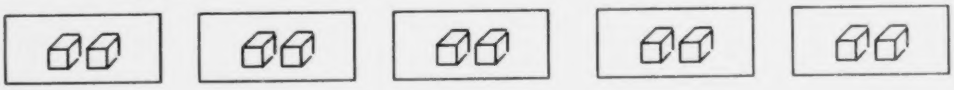
- How many are to be shared?



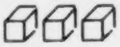
- How many are doing the sharing?



- What is the greatest number that each will have when the sharing is done?



- Are there any left over?

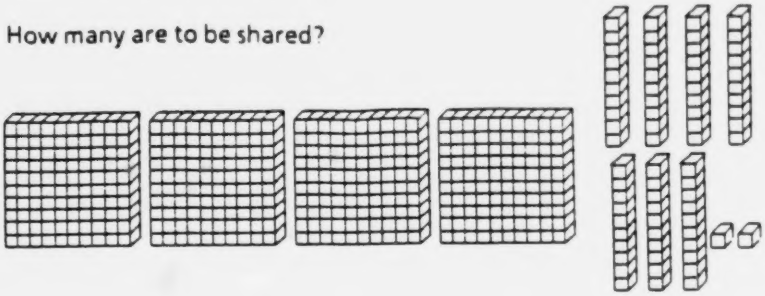


Therefore, 13 ÷ 5 = 2 R 3.

CLARIFICATION/EXAMPLE

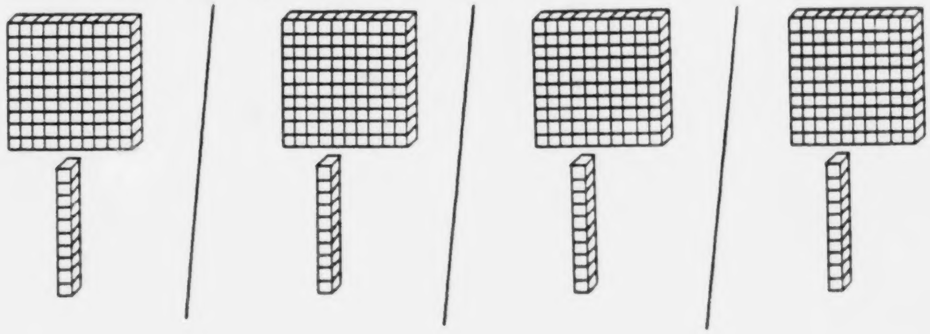
472 ÷ 4 = ?

- How many are to be shared?

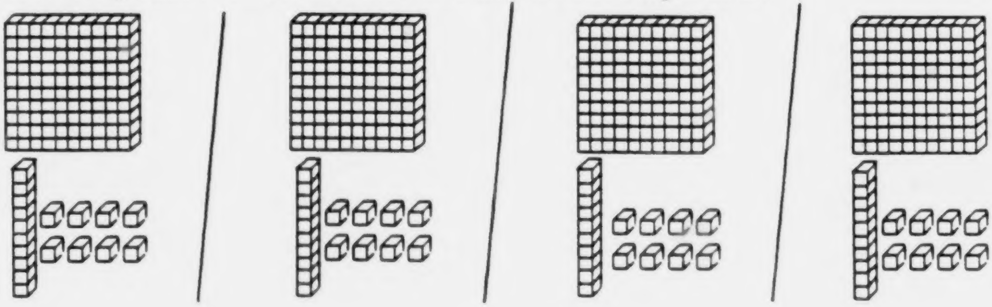


- How many are doing the sharing?

- Can we share so that everyone gets a hundred? a ten?



- What is the greatest number each will have when the sharing is done?



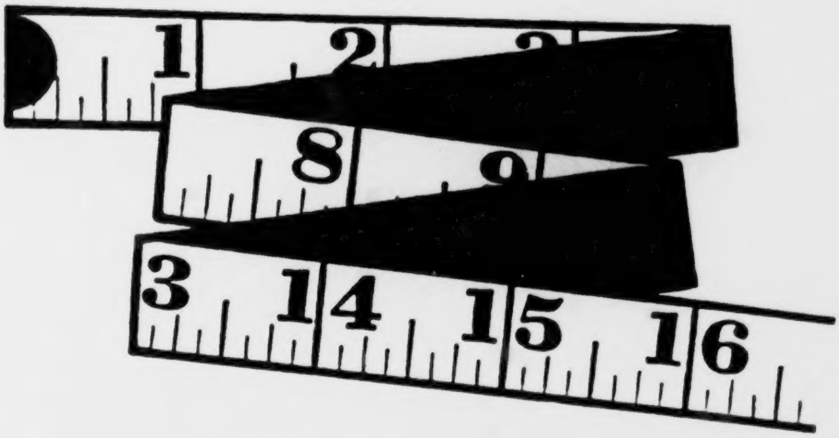
Therefore, 472 ÷ 4 = 118 R 0.

Students may benefit from representation of the division process in a variety of forms. The tape measure provides an alternative method of demonstrating the division process.

CLARIFICATION/EXAMPLE

16 ÷ 3 = ?

Locate the first 16 units on the tape measure. Starting at 16, fold this portion of the tape measure into sections that are 3 units in length.



The number of folded sections will represent the quotient. The number of units left beyond the last fold will represent the remainder.

Therefore, 16 ÷ 3 = 5 R 1.

Some students may experience difficulty in understanding the traditional algorithm for division. If students find other algorithms more meaningful, encourage their use.

CLARIFICATION/EXAMPLE

Alternative Division Algorithms:

```
                                                   4
                                                  40
                                                 200         244
    ________            ________            _________       ________
4 )  978          4 )  978            4 )   978       4 ) 978
     400   100         800   200            800           8
     578               178                  178           17
     400   100         160    40            160           16
     178                18                   18            18
     160    40          16     4             16            16
      18                 2   244              2             2
      12     3
       6
       4     1
       2   244
```

DEVELOPING MENTAL ARITHMETIC SKILLS

Mental arithmetic and estimation have a reciprocal relationship in that the ability to perform simple mental calculations is basic to refining one's estimation skills, and some methods of estimation provide a framework for performing mental arithmetic. Nevertheless, two distinguishing characteristics of mental arithmetic are that it:

- produces an exact answer
- is performed mentally, without the aid of external devices such as paper and pencil.

Teachers can assist students to develop effective mental arithmetic skills throughout the mathematics program by:

- devoting time each day to the development of strategies for mental arithmetic. Plan to teach specific strategies and offer opportunities for practice on a regular basis
- making a list of mental arithmetic strategies that you consider are most appropriate for your students. Encourage discussion and sharing of these strategies
- asking students to "think aloud" and to share the strategies that they already use. In addition to helping students to focus on their own strategies, this process may provide other students with new strategies for performing mental computation
- including the use of whole numbers, decimals and fractions in mental arithmetic activities. Develop strategies for performing various operations with these numbers
- providing frequent opportunities for students to practise mental arithmetic through oral activities. Emphasis on written computational work often tends to discourage the application of skills in mental computation
- developing an evaluation plan that reflects instructional commitment to mental arithmetic. Regular assessment of mental arithmetic skills will focus students' attention on the importance of these skills.

APPLYING BASIC NUMBER SKILLS

Teachers can provide opportunities at the beginning or end of each class for students to apply basic number skills. Questions should be answered without the use of paper and pencil, and may involve:

- single digit products and dividends (e.g., 7×9; $63 \div 7$)
- "extended" products and dividends (e.g., 7×90; $630 \div 7$)
- sequences of operations (e.g., $5 \times 6 + 5 - 7$)
- multiplication and division by powers of ten.

Activities that may assist students to apply basic number skills include:

- the number game "Krypto". This game can be played for short periods of time, and reinforces single digit number skills and sequences of operations.
- "mad-minute" drills. Timed challenges that become part of the daily routine will encourage recall of single digit products and quotients, as well as foster a habit for their use.
- the use of "graph-paper arrays" or "algebra tiles". Students may benefit from constructing arrays that illustrate the factors for "hard to remember" numbers.
- the use of computer programs. A variety of computer programs that reinforce basic number skills are available (e.g., Fast Facts).

Encourage students to "extend" application of their basic number skills to related situations. This approach lends variation to practice sessions, and further develops number sense.

CLARIFICATION/EXAMPLE

- Extended Number Facts for "2 + 3":
  - 20 + 30
  - 20 000 + 30 000
  - 0.2 + 0.3.
- Extended Number Facts for "6 × 7":
  - 6 × 70
  - 60 × 70
  - 0.6 × 700.

Develop strategies for "doubling" and "halving" and demonstrate the application of these strategies in a variety of practical situations involving mental computation.

CLARIFICATION/EXAMPLE

Use a doubling strategy to recall a forgotten multiplication fact, such as 12 × 7.

Step One: Take half of 12 and multiply by 7.
e.g., $6 \times 7 = 42$

Step Two: Double the result.
e.g., $42 \times 2 = 84$

Therefore, $12 \times 7 = 84$.

Factoring is often an effective method of simplifying multiplication, and can assist students to find answers through the process of mental arithmetic.

CLARIFICATION/EXAMPLE

24 × 15 = ?

Step One: Factor the numbers.

$$24 \times 15$$
$$= (4 \times 6) \times (5 \times 3)$$

Step Two: Rearrange the factors.

$$(4 \times 6) \times (5 \times 3)$$
$$= (4 \times 5) \times (6 \times 3)$$

Step Three: Apply basic facts.

$$(4 \times 5) \times (6 \times 3)$$
$$= 20 \times 18$$
$$360$$

Model and discuss the use of appropriate shortcuts for multiplying by 5, 25 and 50.

e.g.,
- to multiply a number by 5, multiply by 10 and divide by 2
- to multiply a number by 25, multiply by 100 and divide by 4
- to multiply a number by 50, multiply by 100 and divide by 2.

Encourage students to identify and share other shortcuts they presently use when performing mental arithmetic.

USING FORMAL STRATEGIES

Two strategies that are frequently used in performing calculations with mental arithmetic include:

- computing from left to right
- compensation.

Model and discuss the use of these strategies on a regular basis. Prompt students to use these and other effective strategies in their daily work.

CLARIFICATION/EXAMPLE

STRATEGIES FOR ADDING WITH MENTAL ARITHMETIC

LEFT TO RIGHT

④8 + ③5

1. Add the tens. — 40 and 30 is 70
2. Add the ones. — 8 and 5 is 13
3. Add the two sums. — 70 + 13 = 83

㊽ + ③5

1. Look at the whole first number. Add to it the tens in the second number. — 48 and 30 is 78
2. Add the ones in the second number to the sum. — 78 + 5 = 83

COMPENSATION

㊽$^{+2}$ + ㉟$^{-2}$

1. Add a number to one addend to make it a multiple of ten. — 48 and 2 is 50
2. Subtract the same number from the other addend. — 35 − 2 is 33
3. Add the two numbers. — 50 + 33 = 83

㊽ + 19 (for use in adding 19, 29, etc.)

1. To the first number add 20 (the next higher multiple of ten). — 48 and 20 is 68
2. Subtract 1 from the sum. — 68 − 1 = 67

CLARIFICATION/EXAMPLE

STRATEGIES FOR SUBTRACTING WITH MENTAL ARITHMETIC

LEFT TO RIGHT

62 – 25

1. From the first number subtract the tens in the second number.

   62 minus 20 is 42

2. Now subtract the ones in the second number.

   42 – 5 = 37

COMPENSATION

$\overset{+5}{62} - \overset{+5}{25}$

1. Add a number to the minuend to make it a multiple of ten.

   25 and 5 is 30

2. Add the same number to the subtrahend.

   62 + 5 = 67

3. Subtract the two numbers.

   67 – 30 = 37

CLARIFICATION/EXAMPLE

STRATEGIES FOR MULTIPLYING WITH MENTAL ARITHMETIC

LEFT TO RIGHT

6×72

1. Multiply the tens in the second factor by the first factor. 6×70 = 420
2. Multiply the ones in the second factor by the first factor. 6×2 = 12
3. Add the two products. 6×72 = 432

COMPENSATION

6×39

1. Round the second factor to the next higher ten. 39 → 40
2. Multiply the tens by the first factor. 6×40 = 240
3. Multiply the difference by the first factor. 40 – 39 = 1
6×1 = 6
4. Subtract the second product from the first one. 240 – 6 = 234

Additional strategies and classroom activities that will assist students to compute using mental arithmetic are provided in *Mental Math in Junior High*.

DEVELOPING STRATEGIES FOR ESTIMATION

Skill in estimation is necessary for effective problem solving and calculator use. Students need to be able to determine if a particular result is precise enough for the purpose at hand, and be alert to the reasonableness of computational results. More often than not, practical problems involve estimations rather than exact numbers. Estimation, not computation, gives answers to everyday questions like "Do I have enough cash to buy groceries?" and "How many hot dogs should I order for the party?".

Thinking skills and problem solving assume important roles in developing estimation skills. It should be recognized that the process of estimation:

- is performed mentally, usually without paper and pencil
- is done quickly
- produces answers that are not exact, but adequate for making necessary decisions
- reflects individual strategies and produces a variety of estimates as answers.

Students should practise estimation in a variety of ongoing activities involving whole numbers, decimals, fractions and percentages. Evaluation should include assessment of students' ability to estimate in a variety of practical situations. The guidelines that follow will assist in creating a classroom climate that establishes the usefulness of estimation and builds student confidence in making estimates.

- Introduce estimation by referring to situations where estimated or rounded counts are used in everyday life. Extend this activity by having students identify numbers used in the newspaper that are exact amounts and those that are estimates:
  - "Over 1500 Fans View Final Home Game"
  - "$200 000 Spent on Recreational Centre"
  - "2/3 of the Athletes Use Brand X".
- Identify other situations where only an estimate is required. Estimation is useful both as a check on the reasonableness of a computed answer and as an end in itself. Many daily situations require only an estimate.
- Use easy examples in early stages of instruction, and avoid emphasis on a high degree of precision in estimates. Students need to be convinced that estimation is easy and develop a desire to use it.
- Focus attention on the language of estimation. By using appropriate phrases, teachers can communicate the spirit of estimation and help students to understand it.

  e.g., - about 12 1/2
  - close to 9
  - just about 15
  - a little less than 3.5
  - between 8 and 9, but probably closer to 8
  - somewhere between 30 and 40.
- Accept a variety of answers. Students need to understand that there is no one "correct" estimate. Any estimate that is reasonably close to the exact answer is valid. If multiple responses are encouraged, students will learn more about the process of estimation.
- Use oral work and group discussion. In the early stages of instruction, students will be tempted to compute the exact answer and adjust it by rounding. By encouraging oral response, anxiety levels will be reduced and students will be more likely to apply estimation strategies. Oral work promotes "doing estimation in your head" rather than with paper and pencil.

- Present situations where students must determine the degree of precision that is required. Estimates can range from rough to precise. Sometimes only a ball-park estimate is required, but at other times it is important to get closer to the exact answer. Through discussion, encourage students to recognize that both ball-park estimates and more precise ones are appropriate, and the choice of which one to use depends on the situation.
- Ask students to identify the number of digits that an answer to a specific computational question should contain. Focusing on the size of the answer provides a quick and useful check for reasonableness.
- Provide activities that require students to:
  - identify estimates that are unreasonable
  - explain why an estimate is an overestimate or an underestimate
  - adjust an initial estimate by changing it to a closer estimate.
- Assess student progress in estimation. A possible evaluation technique involves using an overhead to project a variety of problems. Display each problem individually for a short period of time (fifteen to twenty seconds). Establish scoring intervals for each problem in advance. Take time to discuss selected problems and the strategies that students have used to solve them.

A computer program in BASIC that will provide students with an opportunity to practise their skills in estimation has been included in this section of the manual (see Resource 1: Estimation Game).

FRONT-END ESTIMATION

This method of estimation is useful in addition and subtraction with numbers of three or more digits. The left hand digits are added or subtracted. An estimate for the remaining part of the problem is then determined by "eyeballing" the remaining digits.

$\begin{array}{r}426\\275\\+126\\\hline\end{array}$	a. Add the hundreds →	400 + 200 + 100 = 700
	b. Estimate the sum of tens and ones →	greater than 100 but less than 200
	c. Estimate the total →	between 800 and 900

$\begin{array}{r}626\\-147\\\hline\end{array}$	a. Subtract the hundreds →	600 − 100 = 500
	b. Compare 26 and 47 →	26 is less than 47
	c. Estimate the difference →	The answer is between 400 and 500.

The front-end method can also be used for some multiplication questions.

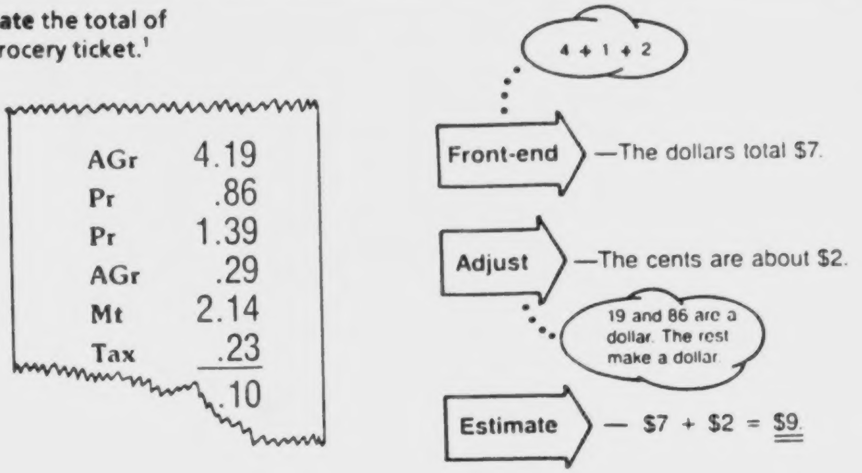
4 X 648 = □

a. 4 X 600 = 2400
b. 48 is about 50
c. 4 X 50 = 200
d. the product is about 2600

One advantage of the front-end method of estimating is that it is less likely to require the use of paper and pencil than other methods.

CLARIFICATION/EXAMPLE

Estimate the total of this grocery ticket.[1]


AGr 4.19
Pr .86
Pr 1.39
AGr .29
Mt 2.14
Tax .23
.10
4 + 1 + 2
Front-end
—The dollars total $7.
Adjust
—The cents are about $2.
19 and 86 are a dollar. The rest make a dollar.
Estimate
— $7 + $2 = $9.

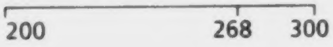
ROUNDING

The concept of rounding numbers needs careful development before students can use it effectively to estimate answers. Students need to understand the thought processes and steps employed in rounding a number. The illustration below uses a number line in developing the concept of a rounded number. Students can apply this thought process to all rounding situations.

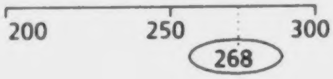
1. Round 268 to the nearest hundred.

268

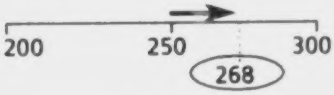
2. Determine which two hundreds the number is between.



3. Find the "midpoint" number.


200
250
300
268

4. Determine which hundred the number is closer to by noting whether it is greater or less than the midpoint number.


200
250
300
268

5. Write the rounded number.

300

Although rounding is the most familiar approach to estimating, it should not be used to the exclusion of other approaches.

---

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1985 by the National Council of Teachers of Mathematics.

CLARIFICATION/EXAMPLE

Example		*Estimate*
362 +825	→	400 +800
49 x6	→	6 x 50
68 x32	→	70 x 30
1782 ÷ 3	→	1800 ÷ 3

COMPATIBLE NUMBERS

Rounding and the use of front-end numbers do not always result in easy mental computation. By rounding, $17.6\overline{)338}$ becomes $18\overline{)340}$. By using front-end numbers, $17.6\overline{)338}$ becomes $17\overline{)330}$. Neither method results in an easy division. Consider, instead, the possibilities of $17\overline{)340}$ or $18\overline{)360}$. Each of these division questions has a quotient that is easy to calculate mentally.

This strategy of estimating with numbers that are easy to calculate mentally is called "estimating with compatible numbers". What constitutes compatible numbers depends upon the operation as well as the estimator's choice.

The compatible numbers strategy can also be used with addition, subtraction and multiplication. As illustrated below, the student looks for pairs of numbers that "fit together" or are easy to calculate mentally.

14.83
7.14 approximately 15 + 25 = 40 40 + 10 = 50
25.82 approximately 3 + 7 = 10
3.41

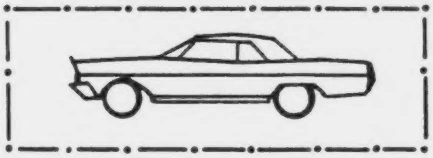
Model this estimation strategy, and prompt students to use the strategy in their daily work.

CLARIFICATION/EXAMPLE

Estimate the monthly payments.[1]

Financed cost: $15 629

Payments: 48 months



The problem is easier to think of as

$$50\overline{)15\,000.}$$

Compatible number estimate: $300

CLUSTERING (OR AVERAGING)

The clustering or averaging strategy is suited to situations where a group of numbers cluster around a common value. To successfully use this strategy, students should understand the concept of "average", and recognize the link between repeated addition and multiplication. Although clustering is limited to particular kinds of problems, the strategy is useful. Many students will discover and use this strategy on their own.

CLARIFICATION/EXAMPLE

Estimate the total attendance.[2]

Monday	72 250
Tuesday	63 891
Wednesday	67 490
Thursday	73 180
Friday	74 918
Saturday	68 490

The figures all cluster around 70 000, so about 70 000 people attended each day.

$$6 \times 70\,000 = 420\,000$$

Averaging estimate: 420 000.

---

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1985 by the National Council of Teachers of Mathematics.

[2] Ibid.

RESOURCE 1: ESTIMATION GAME[1]

The BASIC program which follows provides students with an opportunity to practise their skills in estimation. The program involves addition, but can easily be modified to provide practice with any of the operations. It will work with both whole numbers and decimals.

Players take turns entering a value of their choice. Each player must then make an estimate of the results of adding the numbers that have been entered. The estimate closest to the actual answer is declared the winning estimate.

```
  4  PRINT "ESTIMATION GAME"
 10  LET OP$ = "SUM"
 15  PRINT "IN TURN, EACH PLAYER WILL ENTER"
 20  PRINT "A NUMBER OF THEIR CHOICE"
 30  PRINT "THEN EACH PLAYER WILL ENTER AN"
 35  PRINT "ESTIMATE THE P[$ OF THE NUMBERS"
 45  PRINT "THE CLOSEST ESTIMATE WINS"
 55  PRINT "FIRST PLAYER'S NUMBER": INPUT A
 60  PRINT "SECOND PLAYER'S NUMBER": INPUT B
 70  PRINT "FIRST PLAYER'S ESTIMATE...": INPUT A1
 80  PRINT "SECOND PLAYER'S ESTIMATE..." : INPUT B1
 85  LET C = A + B:  REM FINDS THE SUM
 90  IF ABS (C - A1) < ABS (C - B1) THEN 115
 96  IF ABS (C - A1) = ABS (C - B1) THEN 117
100  PRINT "PLAYER #2 WIND!"
105  GOTO 130
115  PRINT "PLAYER #1 WINS!"
116  GOTO 130
117  PRINT "IT'S A TIE!"
130  PRINT "PLAY AGAIN?  (Y OR N)"
131  INPUT AN$$
132  IF AN$$ = "Y" THEN GOTO 55
133  IF AN$$ < > "N" THEN GOTO 130
140  END
```

Line 10 can be changed to indicate a different operation. By changing line 10 to LET OP$ = "PRODUCT", the instructions in line 35 will tell the user to estimate the answer to a multiplication problem. If line 10 is changed, then the operation in line 85 must also be changed. If OP$ = "PRODUCT", then line 85 must say LET C = A*B.

---

[1] Alberta Education, Curriculum Support, *Junior High Mathematics Teacher Resource Manual*, 1988.

SITUATIONAL AND CONCRETE APPROACHES

Student developmental levels and learning styles suggest an experiential approach to teaching and learning, where students are actively involved in:

- concrete, transitional and formal levels of cognition
- deliberate observation and questioning that promote thinking.

Students need to have direct access to the use of manipulative and visual materials while developing a new concept. Tactile experiences are particularly important in assisting students to deal with cognitive demands of fractions, ratio and percent, as well as two- and three-dimensional geometry. Manipulative materials need not be expensive, and in many instances can be borrowed or improvised. Teachers are encouraged to establish a mathematics lab by borrowing, collecting and/or purchasing a variety of manipulative materials appropriate to the mathematics program. Materials useful in providing instruction at concrete and transitional levels throughout the mathematics program may include:

- assorted measuring devices (e.g., tape measures/rulers/metre sticks, carpenter's squares, measuring cups, graduated cylinders, scales for measuring mass/weight, protractors, compasses, calipers, micrometers)
- assorted mathematical puzzles and games
- base-ten blocks
- cash register tapes and cash summary sheets
- cuisenaire rods
- fraction circles
- geoblocks
- geoboards
- graph paper
- LOGO software
- nails and screws of assorted size
- nuts, bolts and wrenches of assorted size
- paper money and plastic coinage
- specimen bank forms and income tax forms
- tangram pieces
- three-dimensional rectangular solids, cubes, cylinders and cones.

Students also need to observe, question, verbalize and discuss the relationships they investigate at the concrete level, and eventually to translate relationships into the abstract symbols of mathematics. While the purpose of manipulatives is to help students understand and remember, there comes a time when each student should become efficient in making application of concepts in their abstract form. It is important, therefore, that teachers supplement and alternate the use of manipulative materials with other more abstract learning strategies and activities.

Suggestions for planning situational and concrete learning activities in mathematics have been included in this section of the manual and focus attention on:

- Number Systems and Operations
- Ratio, Proportion and Percent
- Geometry and Measurement
- Project Work.

NUMBER SYSTEMS AND OPERATIONS

WHOLE NUMBERS AND DECIMALS

Puzzles and games provide a worthwhile method of reinforcing arithmetical process and basic number skills. Several mathematical puzzles and games have been provided below as examples.

1. 100 GAME

Two players, starting from 0 and taking turns, add any number from 1 to 10. The player who reaches 100 is declared the winner. As students play this game, encourage them to think about a strategy that will ensure their success (i.e., the first player to reach 89 wins the game). Are there other boundary numbers that will also ensure their success?

CLARIFICATION/EXAMPLE

PLAYER 1	PLAYER 2
0	(+9) 9
(+8) 17	(+7) 24
(+9) 33	(+2) 35
(+9) 44	(+5) 49
(+8) 57	(+6) 63
(+7) 70	(+9) 79
(+1) 80	(+9) 89
(+3) 92	(+8) 100
	Player 2 is the winner.

2. LETTER PUZZLES[1]

Students may benefit from designing/solving letter puzzles in addition or subtraction.

In the following puzzle, each letter represents a digit:

$$\begin{array}{r} \text{SEND} \\ \underline{+\text{MORE}} \\ \text{MONEY} \end{array}$$

The solution is:

$$\begin{array}{r} 9\,5\,6\,7 \\ \underline{+\ 1\,0\,8\,5} \\ 1\,0\,6\,5\,2 \end{array}$$

Letter puzzles may also be used to develop and reinforce problem-solving strategies and number properties. In the example that follows, each letter again represents a digit:

$$\begin{array}{r} \text{YZ} \\ \underline{\times\ \ 2} \\ \text{XZZ} \end{array}$$

Ask students to identify the number, which when doubled, yields itself. Students will recognize the multiplicative property of zero, and should then be able to solve the rest of the puzzle.

---

[1] Alberta Education. *Let Problem Solving Be The Focus For The 1980's*, 1983.

The solution is:

$$\begin{array}{r} 50 \\ \underline{\times\ 2} \\ 100 \end{array}$$

Another problem:

$$\begin{array}{r} \mathrm{VZVZ} \\ \underline{\times\quad 2} \\ \mathrm{X2X2Z} \end{array}$$

The solution is:

$$\begin{array}{r} 6060 \\ \underline{\times\quad 2} \\ 12120 \end{array}$$

Relate the use of letters in these puzzles to the use of variables in algebra.

3. PERCENT-DECIMAL DOMINOES (2 PLAYERS)

Shuffle the deck (20 cards) and place cards face down on the table. Each player takes 4 cards from the table.

Play alternates.

Player 1 begins play by placing a domino face up on the table.

Player 2 plays a domino on any edge of the first domino, making sure that the touching edges name the same number. If unable to play, the player draws one card from the face-down cards and plays it if possible. If it is not possible to play, the other person takes a turn.

The first person to play all of their cards wins the game.

Note:
The game may be extended to more than one dealing. Score one point for each card left at the end of each game. The first person to reach 10 points loses the game.

0.07	13%	0.2	80%	0.02	4%	0.9	60%
0.13	8%	0.8	70%	0.04	50%	0.6	31%
0.08	40%	0.7	18%	0.5	6%	0.31	9%
0.4	0.05	0.18	3%	0.06	10%	0.09	1%
5%	20%	0.03	2%	0.1	90%	0.01	7%

4. PERCENT – DECIMAL PUZZLE

Distribute one copy of this grid to each student in the class. Instruct students to cut out the 16 pieces and assemble the puzzle so that touching edges name the same number (i.e., decimal-percent equivalents).

The finished puzzle will be a 4×4 square.

Note:
Other variations of this puzzle are easily made using a 3×3, 4×4 or 5×5 grid.

83% 0.61 0.1 40%	0.04 1% 52% 0.72	0.03 5% 0.8 0.13	23% 0.05 20% 0.79
70% 80% 0.29 0.38	9% 10% 16% 3%	0.61 39% 5% 0.32	0.6 29% 33% 4%
0.06 0.18 8% 61%	72% 17% 0.52 0.31	49% 0.02 81% 60%	13% 0.08 25% 0.23
0.4 0.39 0.05 6%	38% 0.25 0.01 9%	0.09 0.2 0.17 30%	27% 0.16 2% 0.7

5. MAGIC SQUARE

Place the numbers 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 and 1.6 in a square like the one below so that you may add the four numerals in any line and the sum will always be 3.4.

6. SEVEN/FIVE/THREE (2 players)

Lay out fifteen matches as shown.

¶ ¶ ¶ ¶ ¶ ¶ ¶

¶ ¶ ¶ ¶ ¶

¶ ¶ ¶

Players take turns removing matches. Any number of matches can be removed at each turn, providing they are all taken from the same row. The player who takes the last match loses the game.

Students should be encouraged to discover a winning strategy.

INTEGERS

An understanding of integer concepts should be developed by investigating their application in everyday situations. Provide opportunities for students to demonstrate addition of pairs of integers through concrete manipulation and diagramatic representation. An instructional sequence for developing integer concepts is suggested below.

1. Positive and negative integers can be represented to students through the analogy of having money (positive integers) and owing money (negative integers).

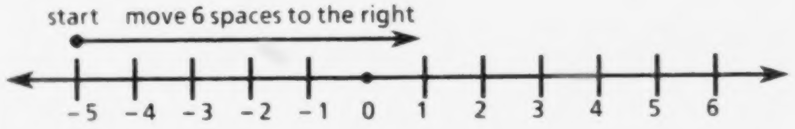
CLARIFICATION/EXAMPLE

Frank makes $7.00 cutting the lawn. He uses this money to pay back a debt of $10.00 which he owes his father. How much money does Frank owe his father after repaying him $7.00?
$-10+7=-3$

2. Integer addition may be represented as moves on the number line. Moves to the right are positive, while moves to the left are negative.

CLARIFICATION/EXAMPLE

$-5+6=?$



Start at −5. Because 6 is positive, move 6 spaces to the right.
$-5+6=1$

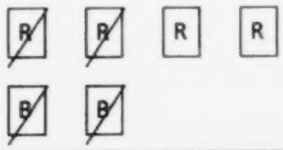
3. Use the thermometer to demonstrate integer addition in the same way that the number line is used. Positive integers represent a rise in temperature, while negative integers will represent a drop in temperature.

4. Coloured cards may be used to demonstrate the addition of integers. Represent positive integers with cards of one colour (e.g., black), and negative integers with cards of another colour (e.g., red).

CLARIFICATION/EXAMPLE

Black cards are positive.
Red cards are negative.

$-4+2=?$



Pairs of red and black cards cancel each other.

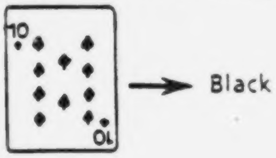
The answer is two red or −2.

5. Integer addition can also be demonstrated using a regular deck of playing cards. Represent positive integers with black cards and negative integers with red cards. Pairs of red and black cards with the same number will cancel each other.

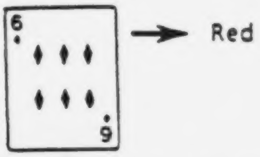
CLARIFICATION/EXAMPLE

$10 + {-6} = ?$

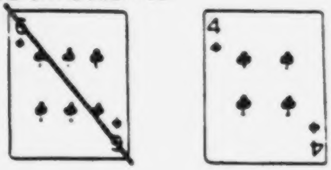
Initially, the problem could be represented as:



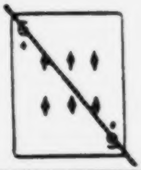
+



These cards do not cancel each other. Replace the 10 with a six and a four. The problem becomes:



+



The answer is a black 4.
$10 + {-6} = 4$

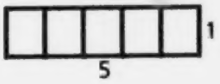
PRIME NUMBERS

The concept of "prime" is abstract, and should be developed using concrete ideas. Some suggestions for developing an understanding of prime numbers are provided below.

1. The following strategy may be effective in developing an understanding of prime numbers:
    - (a) Provide students with a set of square blocks or square tiles. (Pieces of square paper will be sufficient.)
    - (b) Select a number of blocks (square tiles) equal to a number which is to be factored.
    - (c) Form as many rectangles as possible with this number of blocks (square tiles). Record your results.
    - (d) The number being factored is prime if only one rectangle can be formed with the given number of blocks (square tiles).

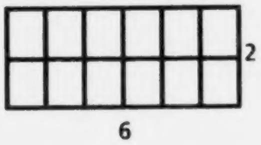
CLARIFICATION/EXAMPLE

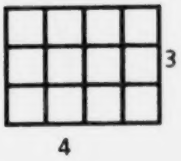
- Factor the number "5":

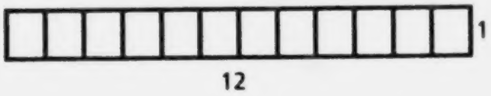


The factors are 1 and 5. Since only one rectangle was formed, the number "5" is prime.

- Factor the number "12":







The factors are 1, 2, 3, 4, 6, and 12. Since more than one rectangle was formed, the number "12" is not prime.

2. Students may not recognize the importance of being able to identify factors and prime numbers. Encourage students to practise these skills by designing games that require their use.

CLARIFICATION/EXAMPLE

List the numbers from 1 to 50.

~~1~~	~~2~~	~~3~~	4	5	~~6~~	7	8	~~9~~	10
11	12	13	14	15	16	17	~~18~~	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

The first player selects a number (e.g., 18) and crosses it out. The second player crosses out all the factors of 18 and scores their total (e.g., 1+2+3+6+9=21). This process is then repeated, with the second player selecting a number and the first player crossing out its factors. Once a number has been crossed out it cannot be used again.

Exchange turns until all numbers have been crossed out. The player with the highest point total is declared the winner.

3. Computer programs can also be used by students to generate sets of factors for numbers they are given.

CLARIFICATION/EXAMPLE

BASIC Program For Generating a Set of Factors

```
 10   PRINT "WHAT IS THE NUMBER"
 20   PRINT "YOU WISH TO FACTOR?"
 30   INPUT N
 40   PRINT "THE FACTORS ARE: "
 50   FOR K = 1 TO N STEP 1
 60   LET X = N/K
 70   LET Y = INT (N/K)
 80   IF X = Y THEN PRINT K
 90   NEXT K
100   END
```

CLARIFICATION/EXAMPLE

BASIC Program For Generating Prime Factors

```
 10   REM LISTS THE PRIME FACTORS
 20   PRINT "ENTER A WHOLE NUMBER"
 30   INPUT N
 40   PRINT "PRIME FACTORS ARE: "
 50   READ P
 60   IF N / P < P THEN 160
 70   IF N / P < P = INT (N / P) THEN 90
 80   GOTO 30
 90   PRINT P
100   LET N = N / P
110   IF N / P < P THEN 160
120   GOTO 70
130   DATA    2, 3, 5, 7, 11, 17, 19
140   DATA    23, 29, 31, 37, 41, 43, 53
150   DATA    59, 61,67, 71, 73, 79, 83, 89, 97
160   PRINT N
170   END
```

4. Prime factors can be used to find the square roots of numbers that are perfect squares. Their use in these situations will assist students to develop concepts of "prime" and "square root".

CLARIFICATION/EXAMPLE

Find the square root of 576.

1. Write 576 as the product of prime factors:
   $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
2. Divide the prime factors into two identical groups:
   $(2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3)$
3. Find the product of each group of factors:
   24×24
4. If $576 = 24 \times 24$, then $\sqrt{576} = 24$.

FRACTIONS

The cognitive demands of concepts related to fractions suggest that instruction include the extensive use of manipulative and visual materials. Select activities that will encourage students to:

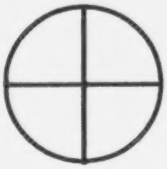
- work with real models, pictures and diagrams
- experiment and explore
- ask questions
- verbalize ideas and discoveries
- make associations and draw conclusions.

Sample activities that provide opportunities for students to manipulate materials, discover relationships and generalize the results of investigation are described below.

1. Fraction circles provide a useful method of developing the concept of a fraction and an understanding of equivalence among fractions. A variety of blackline masters for producing fraction circles have been provided in Resource 1: Fraction Circles.

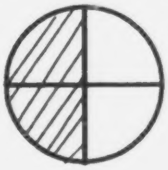
CLARIFICATION/EXAMPLE

- Draw a diagram that illustrates the relationship between 4/4 and 1.



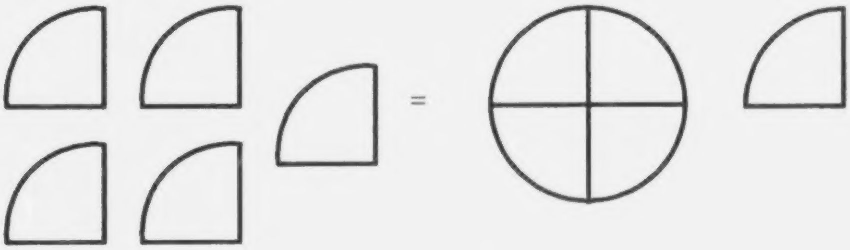
$\frac{4}{4} = 1$

- Draw the diagram that illustrates the relationship between 2/4 and 1/2.



$\frac{2}{4} = \frac{1}{2}$

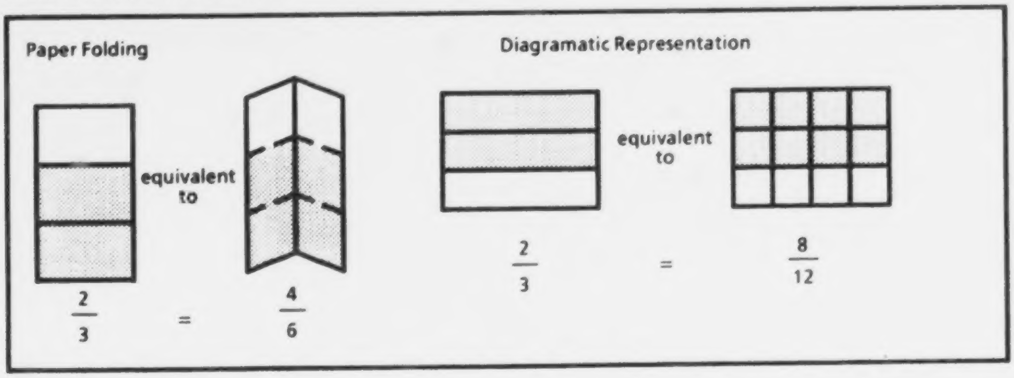
- Investigate the relationship between improper fractions and mixed numbers using pieces of a fraction circle.



$\frac{5}{4} = 1\frac{1}{4}$

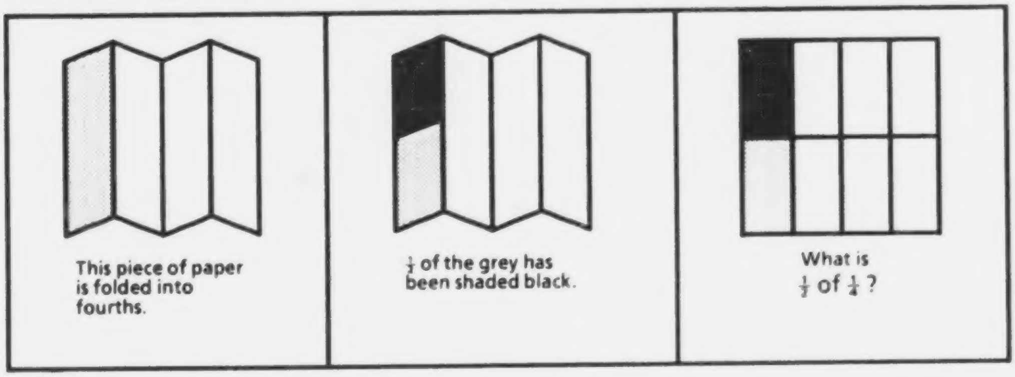
2. The concept of equivalent fractions (ratios) can be developed through paper folding activities, or through diagramatic representation with graph paper/geoboards. Encourage students to translate the relationships discovered into number sentences, and to look for patterns in the ratios obtained.

CLARIFICATION/EXAMPLE


Paper Folding
equivalent to
$\frac{2}{3} = \frac{4}{6}$
Diagramatic Representation
equivalent to
$\frac{2}{3} = \frac{8}{12}$

Paper-folding activities can also be used to demonstrate multiplication of fractions . Encourage students to discover their own strategies for multiplying fractions that are based on the patterns and relationships discovered.

CLARIFICATION/EXAMPLE



This piece of paper is folded into fourths.

$\frac{1}{2}$ of the grey has been shaded black.

What is $\frac{1}{2}$ of $\frac{1}{4}$?

3. The multiple board is a worthwhile tool for investigating equivalent fractions.

CLARIFICATION/EXAMPLE

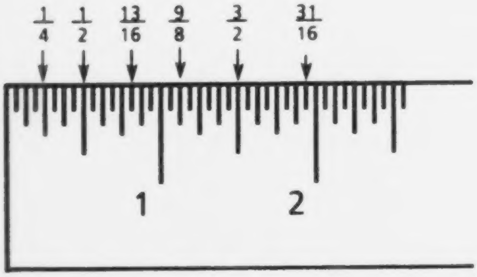
To generate equivalent fractions using the multiple board, locate the original fraction in the first column. Numbers that are in the same position throughout the nine remaining columns will represent fractions that are equivalent to the original fraction.

1	2	3	4	5	6	7	8	9	10
②	④	⑥	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
④	⑧	⑫	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

4. The "inch" unit provides a useful vehicle for developing/reinforcing fraction concepts. Design activities involving fractional parts of an inch that will develop the concepts illustrated below. Black line masters of the inch unit are provided in "Resource 2: Using the Inch Unit".

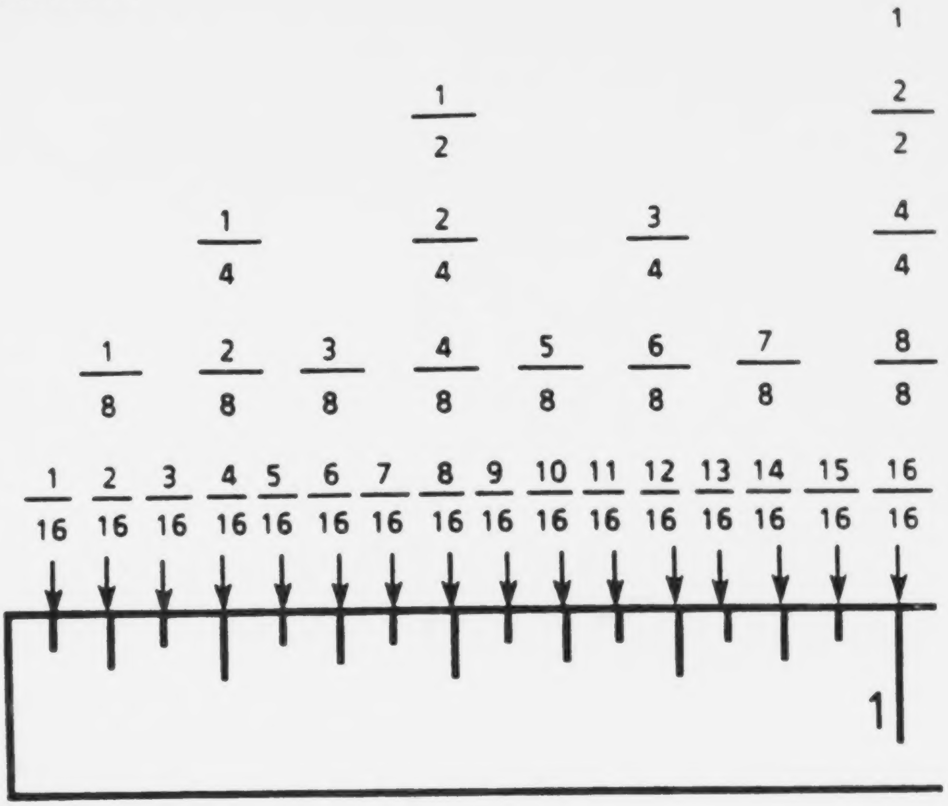
CLARIFICATION/EXAMPLE

Order of Fractions on the Number Line

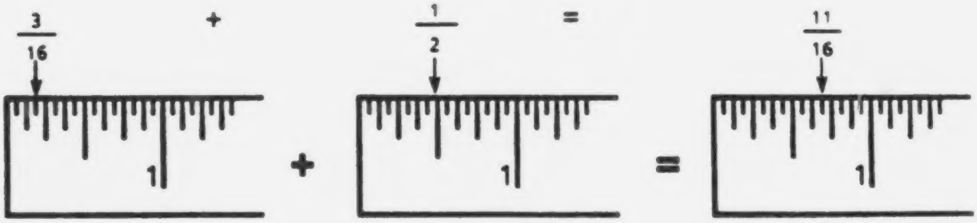


CLARIFICATION/EXAMPLE

Equivalent Fractions


1
1/2 2/2
1/4 2/4 3/4 4/4
1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8
1/16 2/16 3/16 4/16 5/16 6/16 7/16 8/16 9/16 10/16 11/16 12/16 13/16 14/16 15/16 16/16
1

Addition and Subtraction of Fractions


3/16
+
1/2
=
11/16
1
+
1
=
1

5. An understanding of greatest common factors (GCF's) and least common multiples (LCM's) will assist students to simplify and add/subtract fractions. The following examples illustrate alternative methods for finding the greatest common factor and least common multiple of a pair of numbers.

CLARIFICATION/EXAMPLE

What is the GCF of 8 and 12?

Write each number as the product of its prime factors.

$$\begin{array}{ll} 8 & = \textcircled{2} \times \textcircled{2} \times 2 \\ 12 & = \textcircled{2} \times \textcircled{2} \times 3 \end{array}$$

The GCF is the product of all factors common to 8 and 12. Therefore, the GCF is 2 × 2 or 4.

What is the LCM of 8 and 12?

Write the first few multiples of 8 and 12:
8, 16, 24, 32, . . .
12, 24, 36, 48, . . .

The LCM is the smallest multiple that is common to 8 and 12. Therefore, the LCM is 24.

CLARIFICATION/EXAMPLE

An Algorithm for Finding GCF's and LCM's

Divide the numbers by a common factor. The common factor need not be the greatest common factor. Repeat the process until the numbers are relatively prime (i.e., have one as their only common divisor).

$$\begin{array}{r|rr} 2 & 8 & 12 \\ \hline 2 & 4 & 6 \\ \hline & 2 & 3 \end{array}$$

The greatest common factor is found by multiplying the common factors (i.e., numbers on the side).

$$\begin{array}{r|rr} \textcircled{2} & 8 & 12 \\ \hline \textcircled{2} & 4 & 6 \\ \hline & 2 & 3 \end{array}$$

The GCF is 2×2 or 4.

The least common multiple is found by multiplying the common factors (i.e., numbers on the side) by the numbers across the bottom.

$$\begin{array}{r|rr} \textcircled{2} & 8 & 12 \\ \hline \textcircled{2} & 4 & 6 \\ \hline & \textcircled{2} & \textcircled{3} \end{array}$$

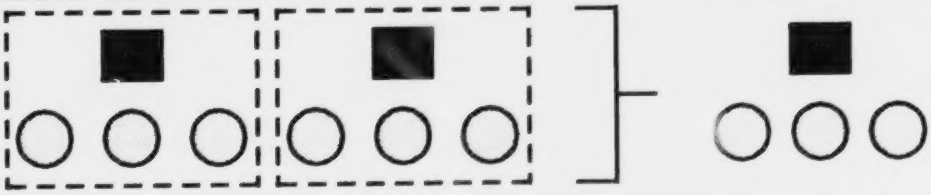
The LCM is 2×2×2×3 or 24.

RATIO, PROPORTION AND PERCENT

It is often assumed that high school students understand the concepts of ratio, proportion and percent because they have been exposed to these ideas throughout their school experience. This may not be the case, and care must be taken to develop these concepts through meaningful activities. Diagnose areas of student strength/weakness, and plan remedial activities that provide concrete and visual experiences for students. Suggestions for developing related concepts and skills are provided below.

1. Encourage students to discover number relationships that are present in equivalent ratios by mapping a correspondence between objects in one collection and objects present in another collection. Through discussion, develop an understanding of how the equivalence of two ratios can be verified using common factors or multiples.

CLARIFICATION/EXAMPLE



$$\begin{tikzcd} \frac{2}{6} \arrow[r, bend left, "\div 2"] \arrow[r, bend right, "\div 2"'] \arrow[r, phantom, "="] & \frac{1}{3} \end{tikzcd}$$

2. Students frequently fail to understand that the relationship between the two parts of a ratio is multiplicative rather than additive. The multiplicative relationship in ratios must be frequently illustrated and explained to students.

CLARIFICATION/EXAMPLE

$\frac{2}{3} = \frac{?}{6}$

Students may incorrectly reason that:

$$\xcancel{\begin{tikzcd} \frac{2}{3} \arrow[r, bend left, "+3"] \arrow[r, bend right, "+3"'] \arrow[r, phantom, "="] & \frac{5}{6} \end{tikzcd}}$$

Emphasize the multiplicative relationship:

$$\begin{tikzcd} \frac{2}{3} \arrow[r, bend left, "\times 2"] \arrow[r, bend right, "\times 2"'] \arrow[r, phantom, "="] & \frac{4}{6} \end{tikzcd}$$

Students who continue to experience difficulty with this concept should be given additional experience in working with collections of real objects, and in mapping a correspondence between objects in one collection with objects in another.

3. Keep number relationships simple when working with ratio and proportion. When applying ratio concepts to practical problems, select problem situations that involve both integral and non-integral ratios:

   - $\frac{1}{3} = \frac{4}{N}$ (integral)
   - $\frac{4}{5} = \frac{N}{15}$ (non-integral).

4. Discuss the use of certain words in making comparisons that can be described with ratios:
   - six books for each student
   - six books per student
   - six for a dollar.

5. When finding the missing component in a proportion, encourage students to use the common factor/multiple method whenever possible.

CLARIFICATION/EXAMPLE

Common Multiple Method

$$\frac{2}{3} \overset{\times 3}{\underset{\times 3}{=}} \frac{N}{9}$$

Numerator and denominator are multiplied by 3.

Common Factor Method

$$\frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{N}{3}$$

Numerator and denominator are divided by 4.

6. Develop formal equation-solving strategies that can be used to find the missing component in proportions not easily solved using the common factor/multiple method. Encourage students to recognize the "cross-product" method of solving proportions as a shortened form of these more formal equation-solving strategies.

CLARIFICATION/EXAMPLE

Find the missing component in $\frac{N}{3} = \frac{1}{4}$.

Formal Strategy

$$\frac{N}{3} = \frac{1}{4}$$

$$12 \times \frac{N}{3} = 12 \times \frac{1}{4}$$

$$\overset{4}{\cancel{12}} \times \frac{N}{\underset{1}{\cancel{3}}} = \overset{3}{\cancel{12}} \times \frac{1}{\underset{1}{\cancel{4}}}$$

$$4 \times N = 3 \times 1$$

$$N = \frac{3}{4}$$

Cross-Product Method

$$\frac{N}{3} = \frac{1}{4}$$

$$\frac{N}{3} \times \frac{1}{4}$$

$$4 \times N = 3 \times 1$$

$$N = \frac{3}{4}$$

7. Assist students to design or modify a simple computer program that will calculate the missing component in a proportion (see Use of Technology, "The Computer").

CLARIFICATION/EXAMPLE

A BASIC Program for Solving Proportions

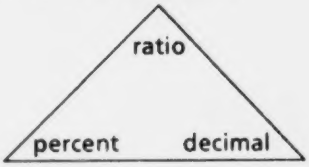
```
 10  REM GENERATE EQUIVALENT
 20  REM RATIOS OF THE FORM
 30  REM A / B = C / D
 40  PRINT "WHAT ARE THE VALUES OF A AND B?"
 50  PRINT "ENTER AS A,B"
 60  INPUT A,B
 70  PRINT "IF C IS GIVEN ENTER C"
 80  PRINT "IF D IS GIVEN ENTER D"
 90  INPUT 0$
100  IF 0$ = "C" THEN 170
110  PRINT "ENTER THE VALUE OF D"
120  INPUT D
130  PRINT "THE VALUE OF C IS"
140  LET C = A * D / B
150  PRINT C
160  GOTO 220
170  PRINT "ENTER THE VALUE OF C"
180  INPUT C
190  PRINT "THE VALUE OF D IS"
200  LET D = C * B / A
210  PRINT D
220  END
```

8. Use a variety of procedures to reinforce understanding of common fraction, decimal and percent equivalents:
   - number line activities
   - fraction-decimal-percent dominoes.

   Triangular drill cards similar to the one illustrated below will be useful in developing student ability to recall equivalents that are commonly used.

CLARIFICATION/EXAMPLE

Construct a set of triangular cards. The vertices of each card will name a ratio, its equivalent decimal, and its equivalent percent.



Encourage students to make their own set of triangular cards for fractions, decimals and percents that they frequently use. Students can cover one or two vertices on each triangle, and practise naming the missing equivalents.

242

9. Students may benefit from producing a table of ratio/decimal/percent equivalents frequently encountered in work situations and everyday life. Encourage students to make such a table and refer to it as necessary.

The following computer program will generate decimal and percent equivalents for ratio "families". A sample run has been illustrated.

CLARIFICATION/EXAMPLE

A BASIC Program for Generating Ratio/Decimal/Percent Equivalents

```
 10  REM THIS PROGRAM WILL GENERATE
 20  REM DECIMAL AND PERCENT EQUIVALENTS
 30  REM FOR RATIO FAMILIES
 40  PRINT "WHAT IS THE DENOMINATOR"
 50  PRINT "OF YOUR RATIO FAMILY?"
 60  INPUT D
 70  FOR K = 1 TO D
 80  PRINT K "/"D, K/D, K/D * 100 "%"
 90  NEXT K
100  END
]RUN
WHAT IS THE DENOMINATOR OF YOUR RATIO FAMILY?

  ?16

 1/16              .0625                 6.25%
 2/16              .125                  12.5%
 3/16              .1875                18.75%
 4/16              .25                     25%
 5/16              .3125                31.25%
 6/16              .375                  37.5%
 7/16              .4375                43.75%
 8/16              .5                      50%
 9/16              .5625                56.25%
10/16              .625                  62.5%
11/16              .6875                68.75%
12/16              .75                     75%
13/16              .8125                81.25%
14/16              .875                  87.5%
15/16              .9375                93.75%
16/16              1                      100%
```

GEOMETRY AND MEASUREMENT

GEOMETRY

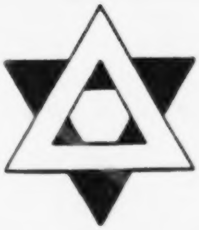
Concepts of one-dimensional space are usually well understood. Students find two-dimensional and three-dimensional concepts more difficult, and may require concrete support for successful experience in these areas. Manipulation and construction will assist students in recognizing the spatial relationships inherent in these figures. A variety of instructional activities that may assist students to develop concepts in geometry and skills in visual perception are provided below.

1. Develop and reinforce an understanding of the properties of two- and three-dimensional shapes by:
   - identifying/discussing the properties of two- and three-dimensional shapes present in the classroom
   - making a list of the two- and three-dimensional shapes observed in a photograph/filmstrip/movie, or after taking a short walk
   - making a collage using various two- and three-dimensional shapes.

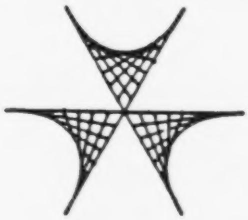
2. Provide opportunities for students to draw and construct the one-, two- and three-dimensional figures being studied using compass/protractor/straightedge/ruler.

   Many geometric concepts and skills can be developed/reinforced as students construct the geometric logos or patterns described in Resource 3: Geometric Patterns. The line design and string art projects provided in Resource 4: Line Design are also useful vehicles for delivering skills in geometry.

   CLARIFICATION/EXAMPLE



Geometric Patterns/Logos



Line Design/String

   Encourage students to be creative in the designs and patterns they produce. Displays of student work in the classroom and hallway will enhance student motivation and effort.

   Additional ideas for geometric patterns, line design and string art projects can be obtained from a variety of resources available on these topics.

3. Use a LOGO computer program to produce geometric figures and designs. A list of LOGO commands is provided in the Use of Technology: The Computer section of this manual. Encourage students to be creative in the designs they generate. Provide direction to students as to which line relationships/geometric figures to incorporate into the designs that are produced.

4. Ask students to create a wallpaper pattern, fabric pattern or linoleum design based on repetition of a line design/geometric pattern that has been constructed.

5. Ask students to construct the 60 attribute shapes that are required to play "Attribute Dominoes". Directions for making the shapes and playing the game are provided in Resource 5: Attribute Dominoes.

6. Provide frequent opportunities for students to solve puzzles that require the recognition of spatial patterns and relationships. Puzzles might involve experimentation with:

- tangrams
- tessellations
- mazes
- paper folding.

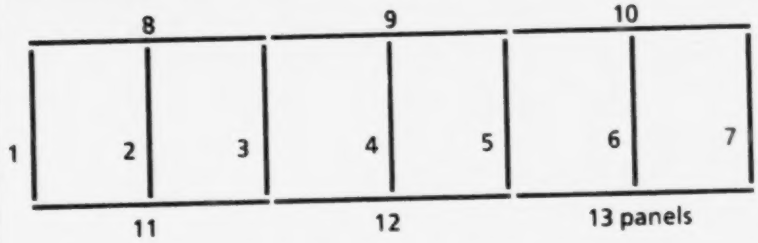
Sample activities involving tangrams and tessellations are provided in Resource 6: The Tangram and Resource 7: Tessellations.

A variety of books containing recreational problems and activities in mathematics can be obtained from local media centres and libraries.

CLARIFICATION/EXAMPLE

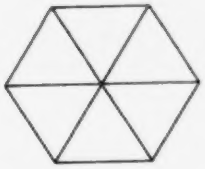
PEN PROBLEM[1]

Six turkey pens were made for a 4H Club display with 13 panels, all of equal length.



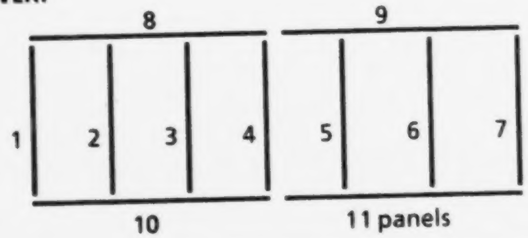
a. One panel was broken when a truck backed into it. The club members then arranged the 12 remaining panels to form 6 pens of equal size and shape. How did they do it?

ANSWER:



b. The manager needed one of the panels in another part of the display. The club members then made 6 pens of equal size and shape with 11 panels. How did they do it?

ANSWER:



---

[1] Alberta Education, *Problem-Solving Challenge for Mathematics*, 1985, p. 47.

7. Introduce students to historical problems and achievements in geometry. Examples might include:

   - The Möbius Strip
   - The Königsberg Bridge Problem
   - Great Pyramids of Egypt
   - The Leaning Tower of Pisa
   - The Theorem of Pythagoras
   - Paintings by M.C. Escher.

CLARIFICATION/EXAMPLE

THE MÖBIUS STRIP

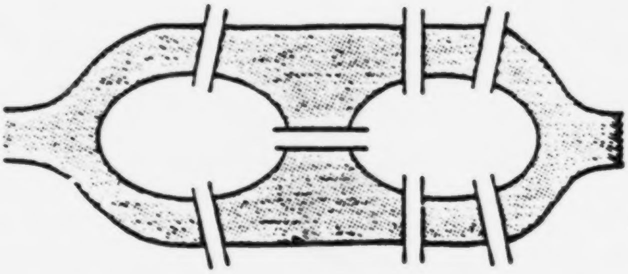
A 19th century mathematician, August F. Möbius, invented what was to become known as the Möbius Strip. The Möbius Strip is a strip of paper that has been given a twist and the ends joined. Some strange results are caused by the twist:

- The strip has no inside or outside. If you draw a line on the strip without raising your pen you will eventually end up where you started.
- If an ordinary strip of paper is cut along the centre you will end up with two identical strips of paper. If you cut the Möbius strip along the centre you will have only one strip twice as long as the original.
- If you cut the strip 1/3 of the width from the edge and continue until it meets up with itself, you will not make the strip longer, but rather will have two interlocking rings.

CLARIFICATION/EXAMPLE

THE SEVEN BRIDGES OF KÖNIGSBERG

This problem originated at Königsberg, a Prussian town on the Preger River, which has seven bridges. The problem was whether it was possible to take a walk and cross each of the bridges exactly once. Try to solve the problem by tracing your path with a pencil.



246

MEASUREMENT

Students need to develop a "feel" for standard units of measure through the use of referents and visual imagery. Provide opportunities for students to develop skill in making reasonable estimates of measure before engaging in activities that involve actual measurement and precision. Consider the following guidelines when planning activities in estimation and measurement.

- Develop skill in estimation by having students make an estimate and then check their estimate through direct measurement.
- Although some experience in direct measurement may immediately follow the introduction of a particular unit, only a few measurements of this nature should be taken before students are asked to make estimates prior to performing direct measurement.
- Encourage good estimates, but do not provide penalties for inaccurate ones.
- The ability to estimate is based on previous experience and must be practised on a regular basis throughout the program.

Students may require concrete support in developing an understanding of the two-dimensional concept of area and the three-dimensional concept of volume. Instruction should include the use of concrete and visual materials (e.g., tiles, grid paper, cubes and models).

Additional suggestions for developing and refining measurement skills are provided below.

1. Provide abundant opportunities for hands-on experience in estimating and measuring the length, mass and capacity of familiar objects.

   One method of developing familiarity with frequently used units is through the use of "mind pictures" or referents. Some examples include:
   - millimetre – thickness of a dime
   - centimetre – width of little finger
   - kilometre – distance from school to __?__
   - gram – weight of a small paper clip
   - kilogram – weight of (subject) textbook.

   Encourage students to use these referents in making estimates, and to check their estimates through actual measurement. Coach students to select units that are appropriate to each item or object being measured. Students can record their work in tables similar to those illustrated.

ITEM	ESTIMATED LENGTH	ACTUAL LENGTH

ITEM	ESTIMATED MASS	ACTUAL MASS

ITEM	ESTIMATED CAPACITY	ACTUAL CAPACITY

2. Another method of estimating measurements is through the use of a strategy called "chunking". The object being measured is divided into portions or chunks whose measure the student is familiar with. The measures of individual portions are then added in order to obtain an estimate of the total measurement. This strategy is particularly effective when the object being measured is large.
   Example: If you wish to measure the height of a room, divide the height into two parts:

   - the height of the doorway
   - the distance between the doorway and the ceiling.

3. Students may also find the strategy of "unitizing" to be an effective method of estimating the measure of a large object. Ask students to divide the object being measured into equal portions. After estimating the measure of one of the equal portions, multiply the estimate by the number of parts into which the object was divided.

4. Encourage students to recognize the importance of measurement in everyday life. Make a display of articles/pictures collected from newspapers and magazines that depict the use of:

   - length
   - perimeter
   - area
   - volume
   - capacity
   - mass
   - time
   - temperature
   - angle.

5. Plan a "scavenger hunt" that will require students to use their skills of estimation and measurement. Divide the class into small groups, and provide each group with the same list of measurements. Each group must go on a scavenger hunt and find objects that have approximately the same measurements as those on the list. No measuring devices may be used. At the conclusion of the hunt, ask each group of students to measure the objects collected. Determine the difference between the estimated and actual measurements. The team with the lowest cumulative total error is declared the winning team.

6. Provide opportunities for students to check the accuracy of their estimates of length/mass/capacity using the activities provided in "Resource 8: Accuracy in Estimation".

7. Develop and reinforce concepts of perimeter, area and volume through the use of tactile and manipulative materials:

   - geoboards
   - dot paper
   - tiles/tessellations
   - boxes/building blocks.

   Activities with the geoboard are provided in Resource 9: Understanding Area.

   Compare the concept of perimeter to "distance around", the concept of area to "surface covered", and the concept of volume to "space occupied".

   CLARIFICATION/EXAMPLE

   Distinguishing Between Perimeter, Area and Volume

   Provide students with a small box. Ask the students to determine:
   - the length of string required to go around the box
   - the amount of paper required to cover the box
   - the number of cm^3 blocks required to fill the box.

   Encourage students to distinguish between perimeter, area and volume in each of these situations. Discuss familiar applications of perimeter, area and volume.

8. Encourage students to develop "mind pictures" for the square centimetre and square metre. Ask students to draw on paper and cut out each of these units.

   Provide opportunities for students initially to verify estimates of area they make by filling rectangular figures with tiles or square cut-outs, and counting the number of squares needed to fill the region. Once the concept of area is understood, encourage students to develop their own formulas for determining the area of familiar geometric figures.

9. Assist students to develop "mind pictures" for the cubic centimetre and cubic metre by making three-dimensional models of these units of measure.

   Provide opportunities for students to make three-dimensional models of rectangular solids and cylinders using the patterns provided in "Resource 10: Three-Dimensional Patterns". Ask students to determine the surface area and volume of the three-dimensional models they make.

10. Students need to develop strategies for using the meters, scales and gauges that they encounter in occupational courses and in everyday life. Mathematics teachers are encouraged to confer with occupational teachers in planning activities that will develop measurement skills that are required by students.

    Students are given practice in interpreting the calibrations on a variety of meters, scales and gauges in "Resource 11: Meters, Scales and Gauges".

PROJECT WORK

Project work in mathematics class often increases student motivation and provides opportunity for the application and maintenance of skills and processes. When effectively planned, a project will integrate mathematical skills with skills learned in other subject areas. Opportunity for group work will foster the development of social skills and responsibility.

Topics around which projects might be developed include:

- Planning a Vacation
- Buying a Car
- Preparing a Personal Time Schedule
- Developing a Budget
- Finding a Place to Live
- Orienteering in the Outdoors
- Operating a Simple Retail Business
- Decorating a Home.

SAMPLE PROJECT: BUYING A CAR

Assume that you are going to buy a car. After considering your monetary resources, decide whether you will buy a new car or used car. Obtain information on buying, financing and maintaining a car from a variety of sources:

- the newspaper
- local financial institutions
- the insurance bureau.

OUTLINE

A. Assessing Personal Finances

- How much money do you have at present?
- How much money do you earn?
- How much money can you save before you buy the car? (Prepare a personal budget.)
- How much money can you afford to spend on a car?

B. Considering Additional Expenses

- Insurance:
  - What type of insurance do you require?
  - How much coverage do you need?
  - What deductible will you need?
- Registration and licencing fees.
- Maintenance and operating costs.

C. Shopping for a Car

- Consumer reports
- Newspaper ads

- Bartering:
  - What does "book value" mean?
  - How do prices compare among competing businesses?
  - What constitutes a reasonable offer?

- Questions to ask:
  - What is the odometer reading?
  - What options does the car have?
  - Was the car in previous accidents?
  - Is the car in good condition? (e.g., Is the car mechanically safe? What is the condition of the car's exterior?)
  - Are there any liens against the car?

D. Financing

- Amount of down payment.

- Value of trade-in.

- Amount to be borrowed.

- Choice of lending institution.

- Type of loan:
  - interest rate
  - payment schedule
  - penalty for late or missed payment
  - amortization period.

E. Driver Responsibilities

- Safe driving practices:
  - driving lessons
  - safe stopping distances
  - safe following distances
  - use of seat belts.

- Routine safety checks:
  - brakes
  - tires
  - lights and turning signals
  - steering and alignment.

INTEGRATED SKILLS

Skills from mathematics and other subject areas that might be integrated with this project are identified below.

Mathematics:

- problem solving
- estimation
- calculator skills
- reading and interpreting tables and charts
- personal budgeting skills
- calculation of interest
- calendar skills
- ratio, proportion and percent skills.

English

- gathering information
- reading and interpreting information presented in the newspaper
- considering the opinions of others
- interpreting the presenter's intent
- writing business letters.

Science:

- recognizing the subsystems in an automobile
- describing simple maintenance procedures that will contribute to vehicle efficiency and safety
- identifying the impact of technological developments on the automobile industry
- using problem-solving and decision-making skills.

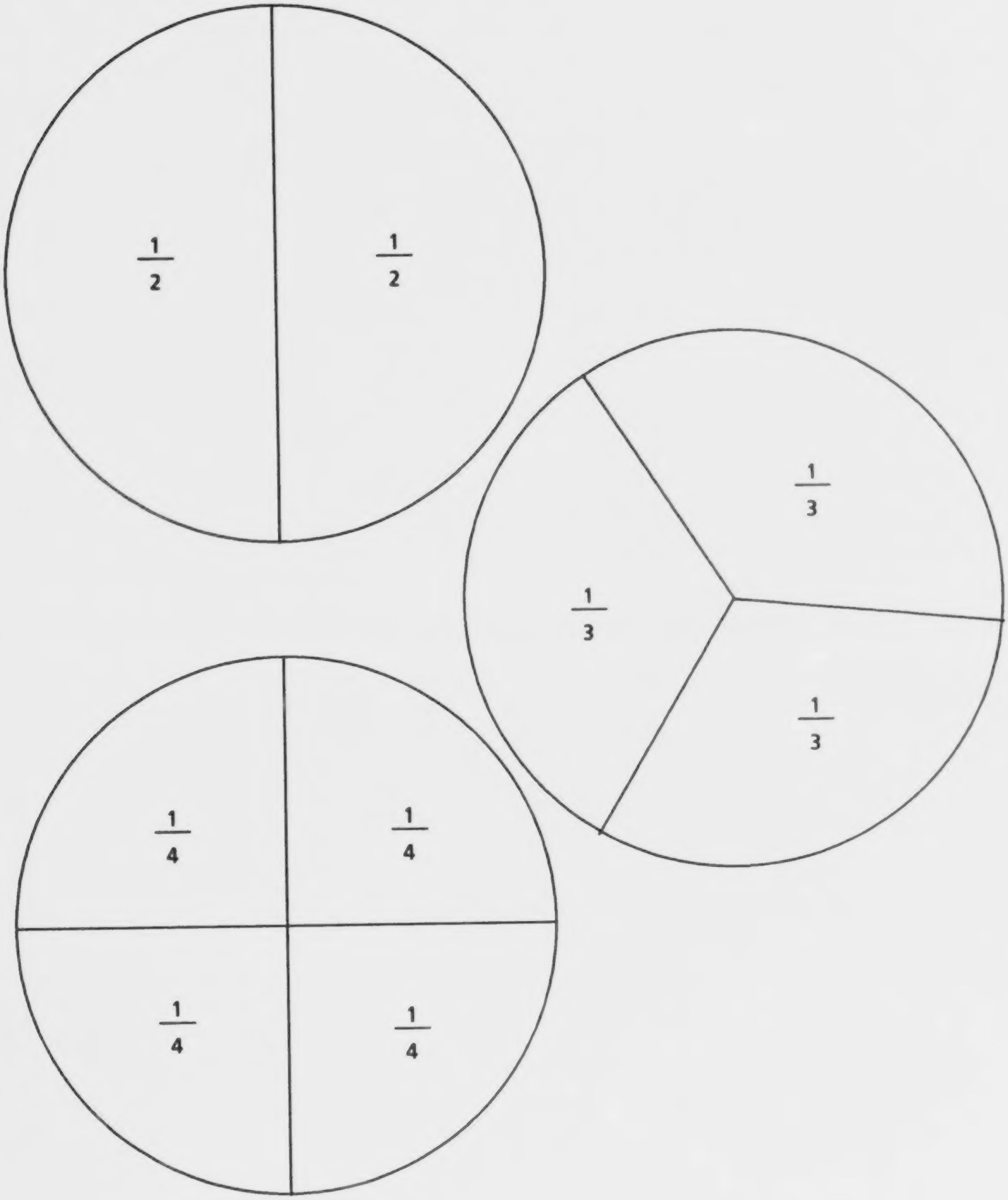
Social Studies:

- decision making
- personal development and personal organization
- interpretation of laws and bylaws
- personal economics
- knowledge of resources in the local community.

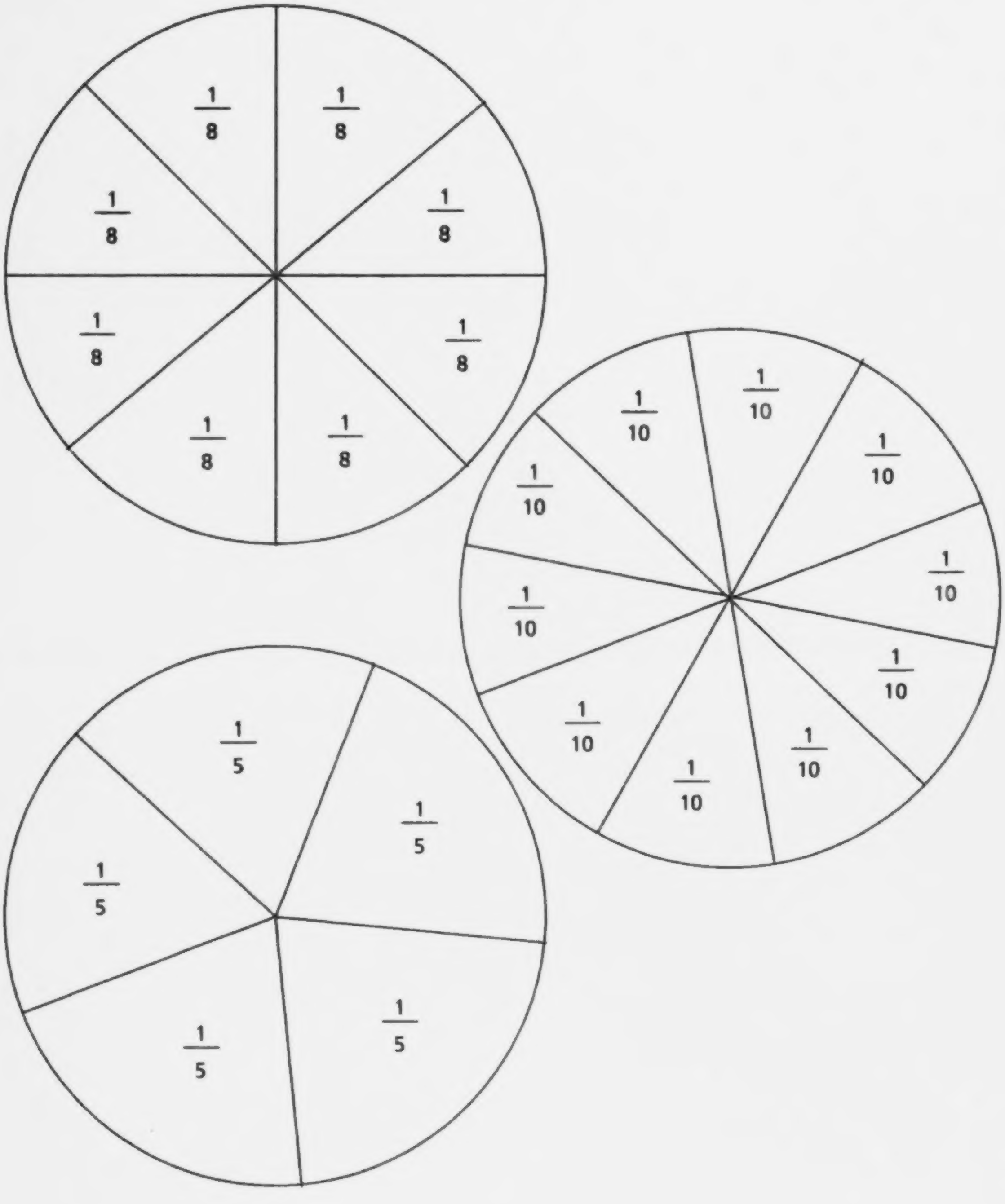
Occupational Courses

- budgeting
- consumer awareness and skills
- personal and interpersonal skills
- business communication skills
- entrepreneurial awareness.

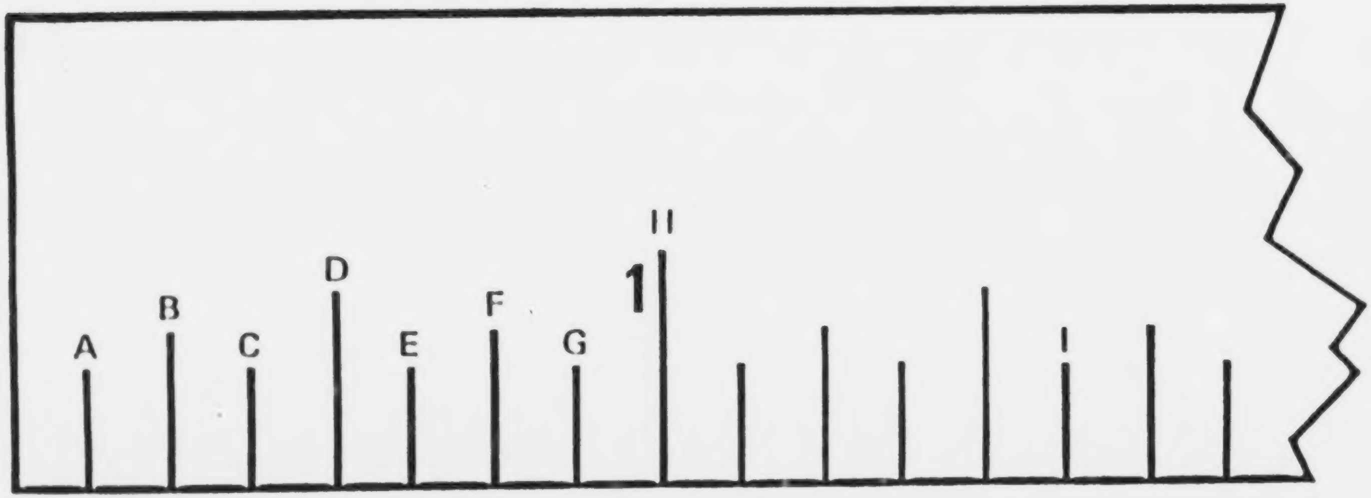
RESOURCE 1: FRACTION CIRCLES


1/2
1/2
1/3
1/3
1/3
1/4
1/4
1/4
1/4

RESOURCE 1: FRACTION CIRCLES (continued)

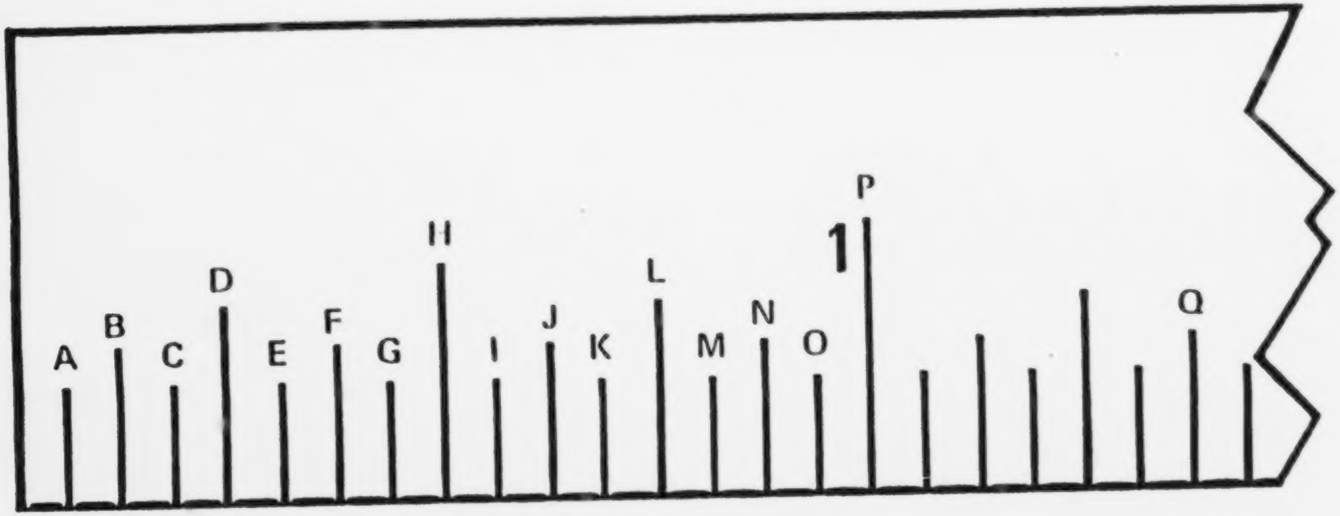

1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/5
1/5
1/5
1/5
1/5

Eighths Rule



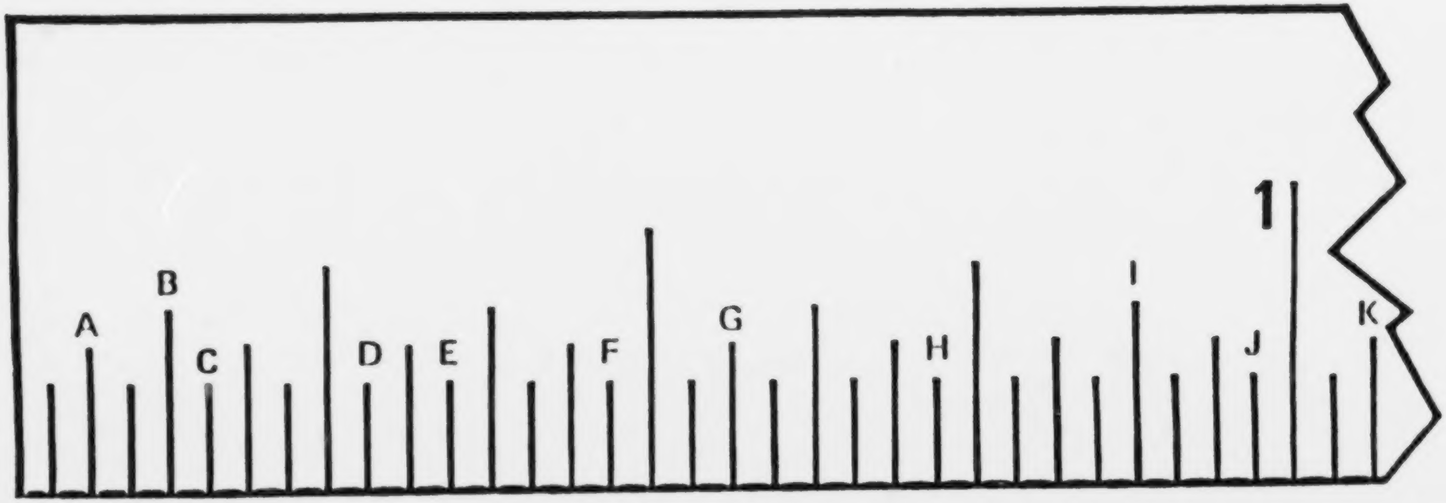
A = 1/8, B = 2/8 = 1/4, C = 3/8, D = 4/8 = 1/2,
E = 5/8, F = 6/8 = 3/4, G = 7/8, H = 8/8 = 1,
I = 13/8 = 1 5/8

Sixteenths Rule



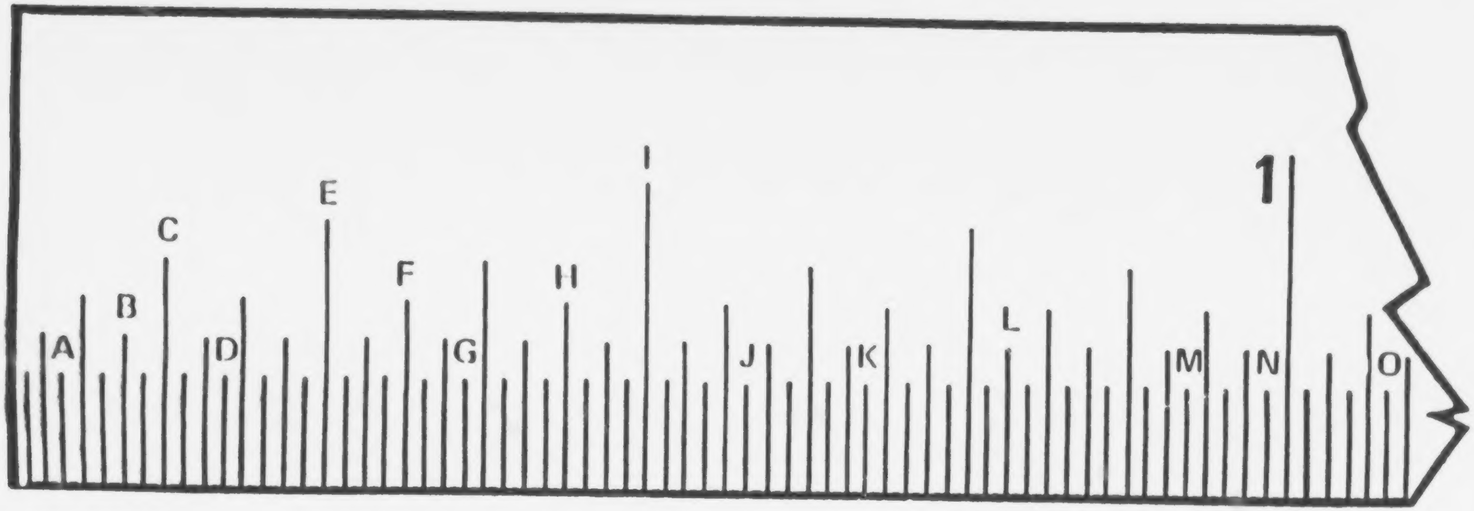
A = 1/16, B = 2/16 = 1/8, C = 3/16, D = 4/16 = 1/4,
E = 5/16, F = 6/16 = 3/8, G = 7/16, H = 8/16 = 1/2,
I = 9/16, J = 10/16 = 5/8, K = 11/16, L = 12/16 = 3/4,
M = 13/16, N = 14/16 = 7/8, O = 15/16, P = 16/16 = 1,
Q = 22/16 = 1 6/16 = 1 3/8

Thirty-Seconds Rule



A = 2/32 = 1/16, B = 4/32 = 1/8, C = 5/32, D = 9/32,
E = 11/32, F = 15/32, G = 18/32 = 9/16, H = 23/32,
I = 28/32 = 14/16 = 7/8, J = 31/32,
K = 34/32 = 1 2/32 = 1 1/16

Sixty-Fourths Rule



A = 3/64, B = 6/64 = 3/32, C = 8/64 = 1/8, D = 11/64,
E = 16/64 = 8/32 = 1/4, F = 20/64 = 10/32 = 5/16,
G = 23/64, H = 28/64 = 14/32 = 7/16, I = 32/64 = 1/2,
J = 37/64, K = 43/64, L = 50/64 = 25/32, M = 59/64,
N = 63/64, O = 69/64 = 1 5/64

RESOURCE 3: GEOMETRIC PATTERNS

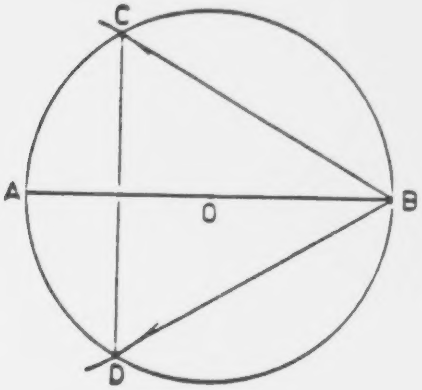
The constructions of inscribed equilateral triangles, regular hexagons, squares and regular octagons can be used to produce a variety of geometric designs.

A. TRIANGLES AND HEXAGONS

Follow the instructions for inscribing an equilateral triangle and a regular hexagon in a circle. By joining various points and colouring or shading parts of the figures, create geometric designs similar to those illustrated in the examples.

1. Inscribing an Equilateral Triangle

a) With a compass draw a circle with centre 0.
b) In this circle draw a diameter AB.
c) With centre A and radius A0, draw an arc intersecting the circle at C and D.
d) Connect B, C, and D to form an equilateral triangle.

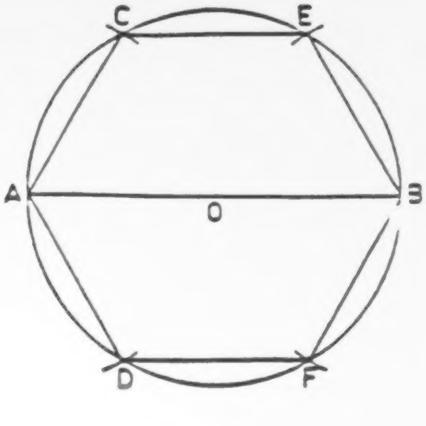


2. Inscribing a Regular Hexagon

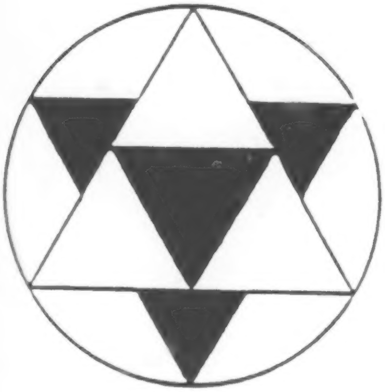
a) With a compass draw a circle with centre 0.
b) In this circle draw a diameter AB.
c) With centre A and radius A0, draw arcs intersecting the circle at C and D. Similarly, with centre at B and radius B0, draw arcs intersecting the circle at E and F.
d) Connect the six points on the circle to form a regular hexagon.

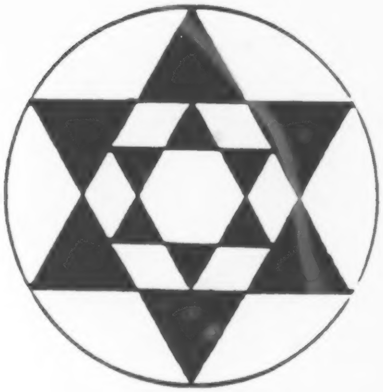
(see diagram on next page)

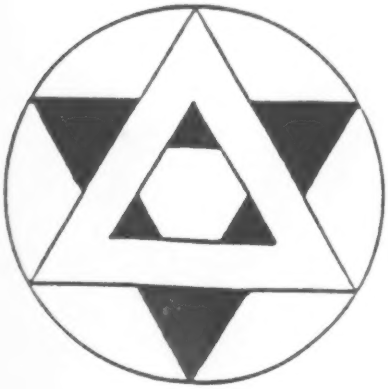
RESOURCE 3: GEOMETRIC PATTERNS (continued)

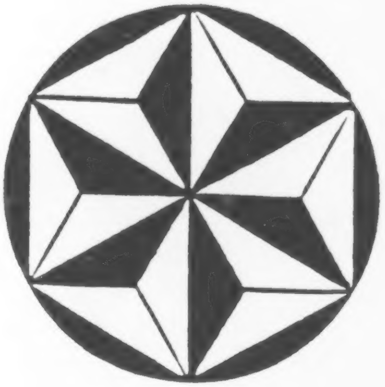


3. Examples of Designs







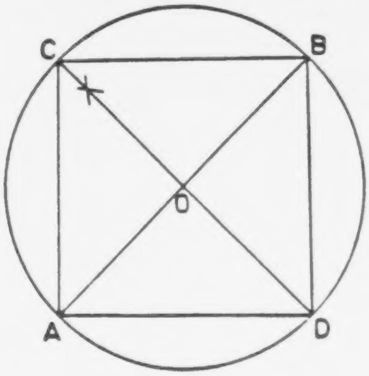


B. SQUARES AND OCTAGONS

Follow the instructions for inscribing a square and a regular octagon in a circle. By joining various points and colouring or shading parts of the figures, create geometric designs similar to those illustrated in the examples.

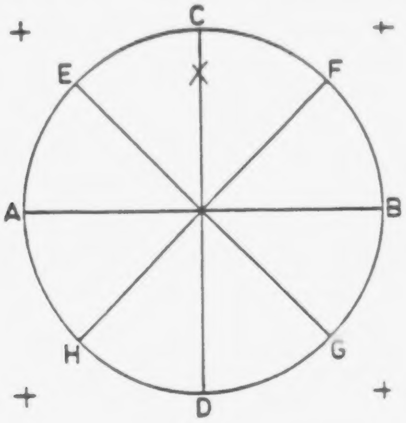
1. Inscribing a Square

a) Draw a circle with centre 0 and draw a diameter AB.
b) Construct another diameter CD which is the perpendicular bisector of AB
c) Connect the four points on the circle to form a square.

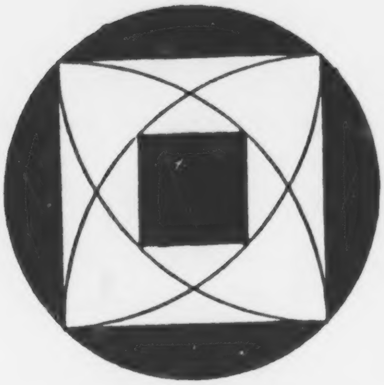


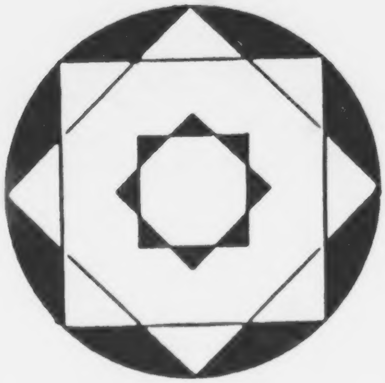
2. Inscribing a Regular Octagon

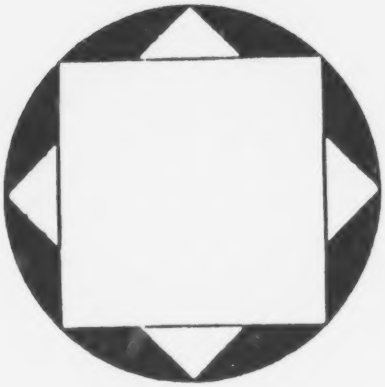
a) Draw a circle with centre 0 and locate points A, B, C and D as in the construction of a square above.
b) Bisect ∠AOC, ∠COB, ∠BOD, ∠DOA to obtain points E, F, G and H on the circle respectively.
c) Connect A, E, C, F, B, G, D and H, in order, to form a regular octagon.

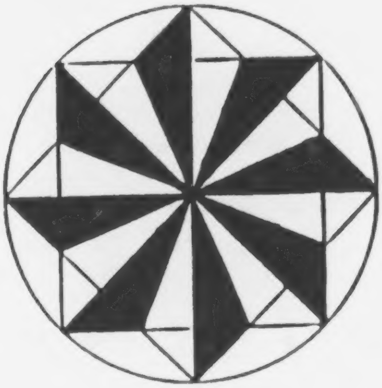


3. Examples of Designs





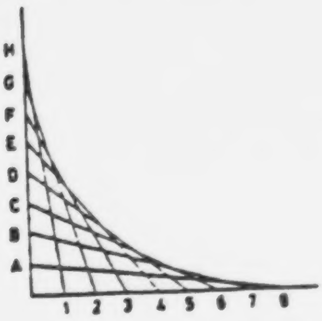




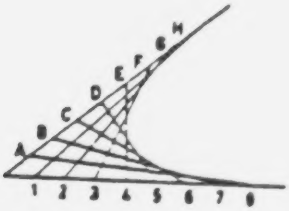
RESOURCE 4: LINE DESIGN

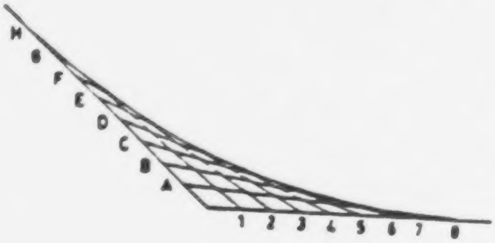
The construction of line segments can produce interesting geometric patterns and designs.

A. Draw a right angle and mark off from the vertex eight segments of equal length along each arm of the angle. (Use either a compass or a ruler to obtain equal lengths.) Label the end points of the segments as in the following diagram and connect A to 8, B to 7, . . . , and H to 1.

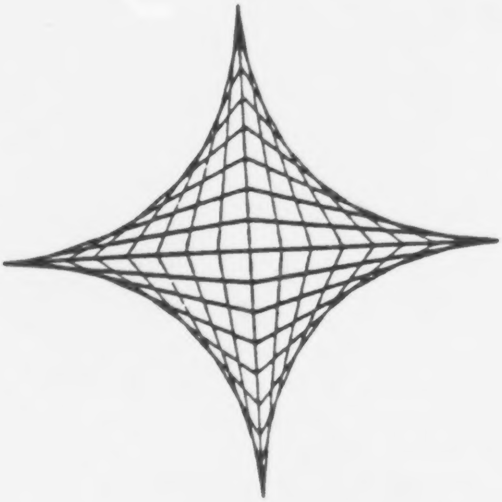


B. Try the procedure outlined above with an acute angle and an obtuse angle.

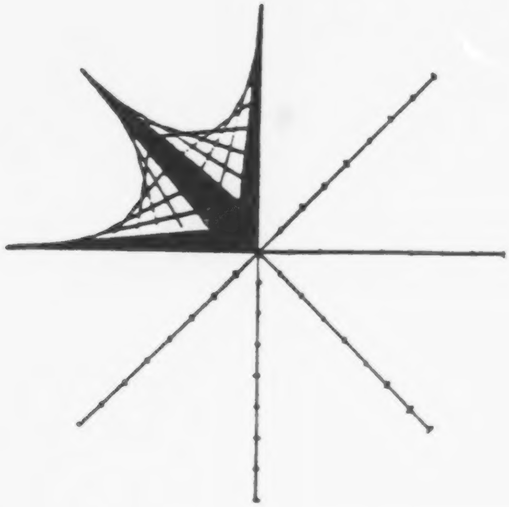




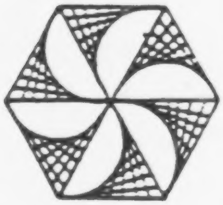
C. Draw a line segment. Construct a second line segment with the same length as the first in such a way that each segment is a perpendicular bisector of the other. Use the four angles formed to create this design.

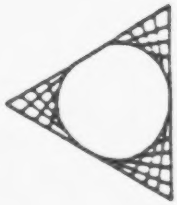


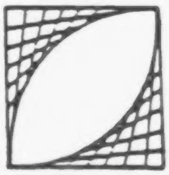
D. More involved designs can be obtained by bisecting angles and using the angles formed for additional designs. The following example makes use of angle bisectors as well as some shading.

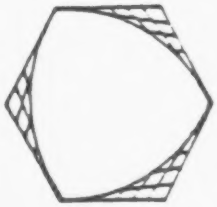


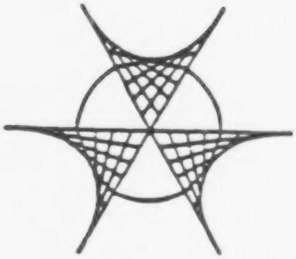
E. Construct additional line designs using the procedures you have learned. The illustrations below may give you some ideas.

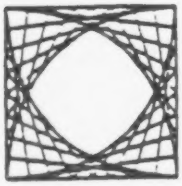


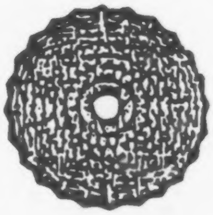


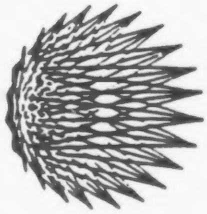










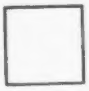


RESOURCE 5: ATTRIBUTE DOMINOES

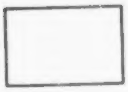
MATERIALS:

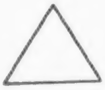
Construct a set of 60 attribute shapes using:

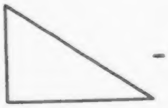
- three different colours of paper (e.g., blue, green, red)
- five large geometric shapes


– large square

– large circle


– large rectangle


– large equilateral triangle


– large isosceles-right triangle

- five small geometric shapes

– small square

– small circle

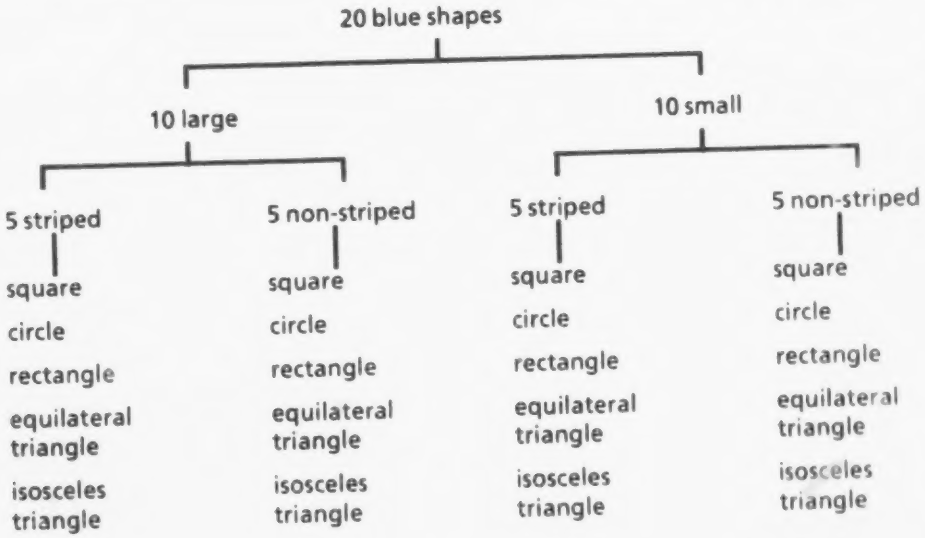
– small rectangle

– small equilateral triangle

– small isosceles right triangle

- striped and non-striped shapes in each size and colour.

The set of 60 attribute shapes will consist of 20 blue shapes, 20 green shapes and 20 red shapes. The chart below illustrates the attributes that will be common to each set of coloured shapes.

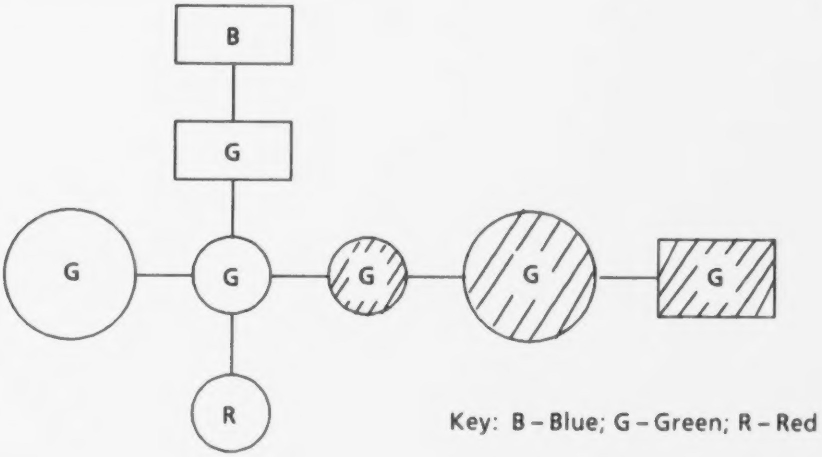

20 blue shapes
10 large
10 small
5 striped
square
circle
rectangle
equilateral triangle
isosceles triangle
5 non-striped
square
circle
rectangle
equilateral triangle
isosceles triangle
5 striped
square
circle
rectangle
equilateral triangle
isosceles triangle
5 non-striped
square
circle
rectangle
equilateral triangle
isosceles triangle

RESOURCE 5: ATTRIBUTE DOMINOES (continued)

DIRECTIONS FOR PLAY:

The game can be played with two to four players, each of whom receives ten attribute shapes. The first player places one game piece in the centre of the table. The other players take turns placing a piece in one of four directions, building from the previous piece. In order to play an attribute shape, it must differ from the piece it is placed beside by only one attribute. If a player cannot play, he must draw a new attribute shape from a container holding the extra pieces. If the player cannot play this shape, the play passes to the next person. The first player to play all of their attribute shapes is declared the winner.

A sample game pattern is illustrated below:



Key: B – Blue; G – Green; R – Red

NOTE: Attribute shapes can be varied according to the geometric figures that are being studied. Students in Grade 9 may wish to use the parallelogram, hexagon and octagon in conjunction with two other geometric shapes.

270

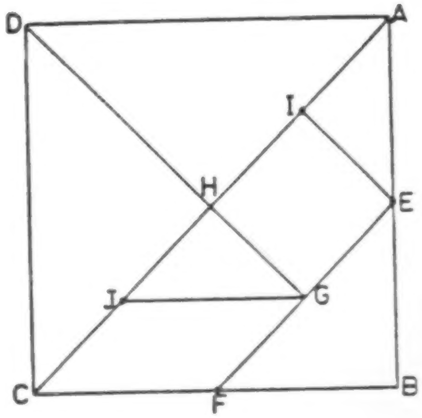
RESOURCE 6: THE TANGRAM

This exercise involves making a variety of geometric figures by rearranging pieces of a puzzle.

MAKING THE PUZZLE

If tangram pieces are not available, a set may be constructed by following these directions:

1. On cardboard draw a square ABCD with each side at least 10 cm in length.
2. Locate midpoints E and F of sides $\overline{AB}$ and $\overline{BC}$ respectively.
3. Draw $\overline{EF}$ and $\overline{AC}$.
4. Draw the portion of $\overline{DB}$ from D to $\overline{EF}$ (shown in the diagram as $\overline{DG}$). Let H be the point of intersection of $\overline{DG}$ and $\overline{AC}$.
5. Locate I, the midpoint of $\overline{AH}$, and J, the midpoint of $\overline{CH}$. Draw $\overline{EI}$ and $\overline{GJ}$.
6. Cut out the seven pieces.



SOLVING THE PUZZLE

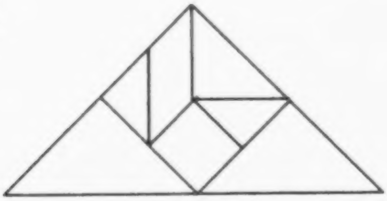
Using the seven tangram pieces, arrange them in such a way as to form:

1. a square
2. a triangle
3. a rectangle which is not a square
4. a parallelogram which is not a rectangle
5. a trapezoid
6. a convex polygon (a polygon in which each of the diagonals fall within its interior; 13 such polygons can be formed with the tangram pieces).

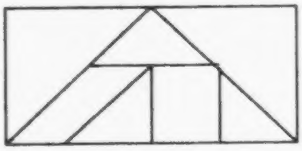
RESOURCE 6: THE TANGRAM (continued)

SOME POSSIBLE SOLUTIONS

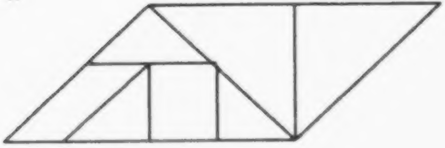
2.



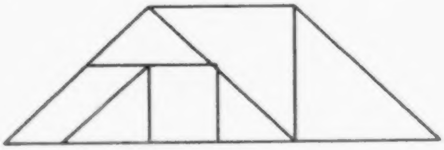
3.



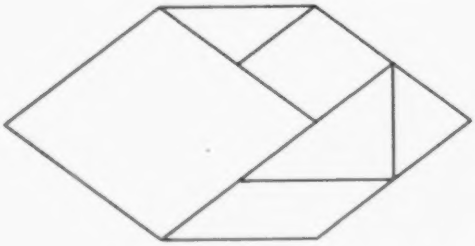
4.



5.



6. One convex polygon can be made in this way. See if you can make the other 12 convex polygons. Record your results.



FINDING THE AREA

Give the small square, EGHI, an area of one square unit. Compute the area of each of the other six tangram pieces using the square as the unit of measure.

RESOURCE 7: TESSELLATIONS

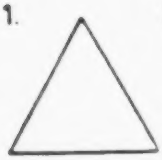
A. A tessellation is a pattern that can completely "fill" a surface or region without overlapping or leaving spaces. The word "tessellation" comes from an ancient Latin word which means "to cover with tiles". Some tessellations which we encounter in our everyday life are tiled floors and wall papers. Name some other examples of tessellations.

B. Construct tiles identical to those shown in Part C using a piece of cardboard. You will require several tiles of each shape. Your tiles will include the following shapes:

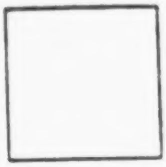
- equilateral triangle
- isosceles right triangle
- parallelogram
- rectangle
- square (three different sizes)
- rhombus
- trapezoid
- regular hexagon.

C. Using only the tiles that have the shape of an equilateral triangle or an isosceles right triangle, cover each of the following shapes. Give the name of the polygon you have covered.

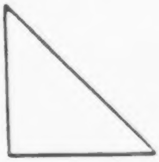
1.



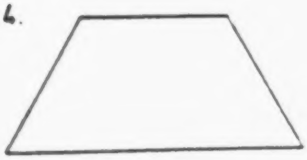
2.



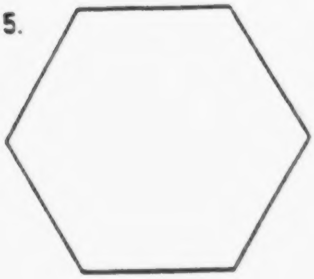
3.



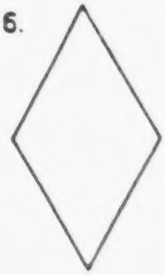
4.



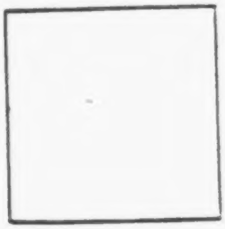
5.



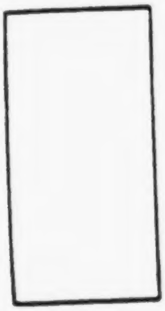
6.



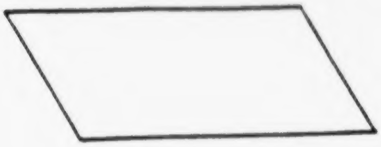
7.



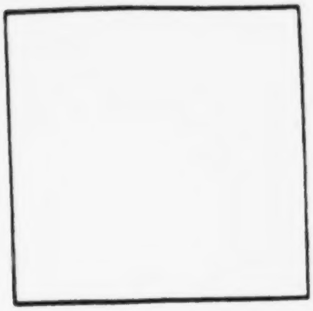
8.



9.



10.



D. Make a sketch to illustrate your use of equilateral triangles to form the hexagon in C5.

Make another sketch to show how a large equilateral triangle can be made from several small ones.

E. Which of the following shapes from Part C can produce tessellations?

	Yes	No
square	_____	_____
parallelogram	_____	_____
rhombus	_____	_____
trapezoid	_____	_____

Sketch the tessellations produced by each of these shapes.

In each of these tessellations, can you find a larger version of the basic shape used to make the tessellation? If you can, outline it on your sketch. Which shapes can produce tessellations but cannot be put together to make larger versions of themselves?

RESOURCE 8: ACCURACY IN ESTIMATION

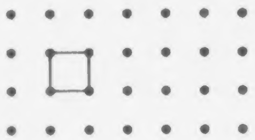
Select a variety of objects in the classroom to be measured. Estimate the measure of each object. Then measure the objects using a variety of measuring tools. Use the formula provided below to express the accuracy of each estimate as a percent.

$$\frac{\text{estimated measure}}{\text{actual measure}} \times 100.$$

OBJECT	ESTIMATE	ACTUAL	ACCURACY
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

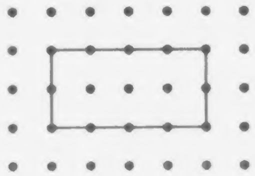
RESOURCE 9: UNDERSTANDING AREA

You will require the use of a geoboard or dot paper in doing these exercises. The unit square illustrated below will be used in measuring the area of a variety of geometric figures.



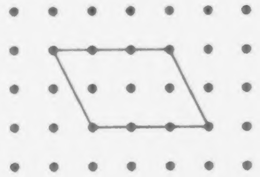
A. RECTANGLE

Form a rectangle on the geoboard. Determine its area by counting the unit squares. Compare the area of the rectangle to its perimeter.

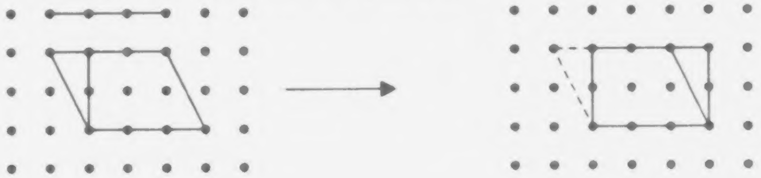


B. PARALLELOGRAM

Form a parallelogram and find its area.



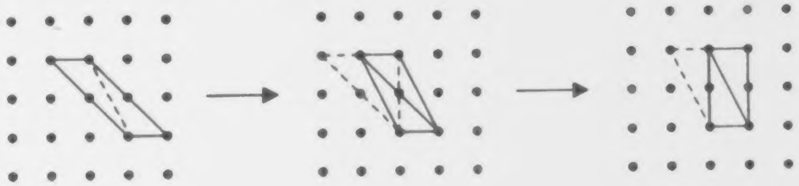
One technique for finding the area is to identify a triangular region within the parallelogram and perform a slide upon it as in the following diagram.



RESOURCE 9: UNDERSTANDING AREA (continued)

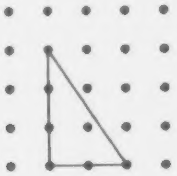
The area of the rectangle can be found by counting. Therefore, the area of the parallelogram is also determined.

In certain cases, this technique has to be applied more than once.

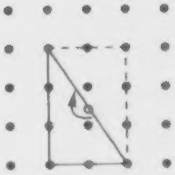


C. RIGHT TRIANGLE

Form a right triangle and determine its area.



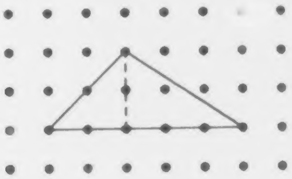
If the triangle is given a half-turn about the midpoint of the side opposite the right angle, a rectangle is identified. The area of the rectangle can be determined by counting. The area of the triangle is one-half the area of the rectangle.



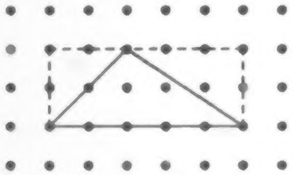
D. SCALENE TRIANGLE

Form a scalene triangle and find its area. Some hints for finding the area are:

1. "Break up" the triangle into right triangles.

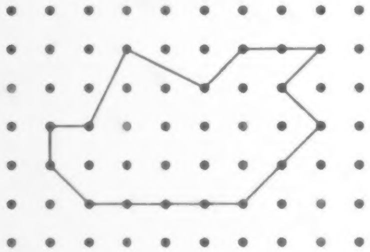


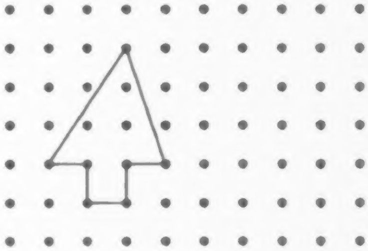
2. Subtract from the area of a larger rectangle.



E. OTHER POLYGONS

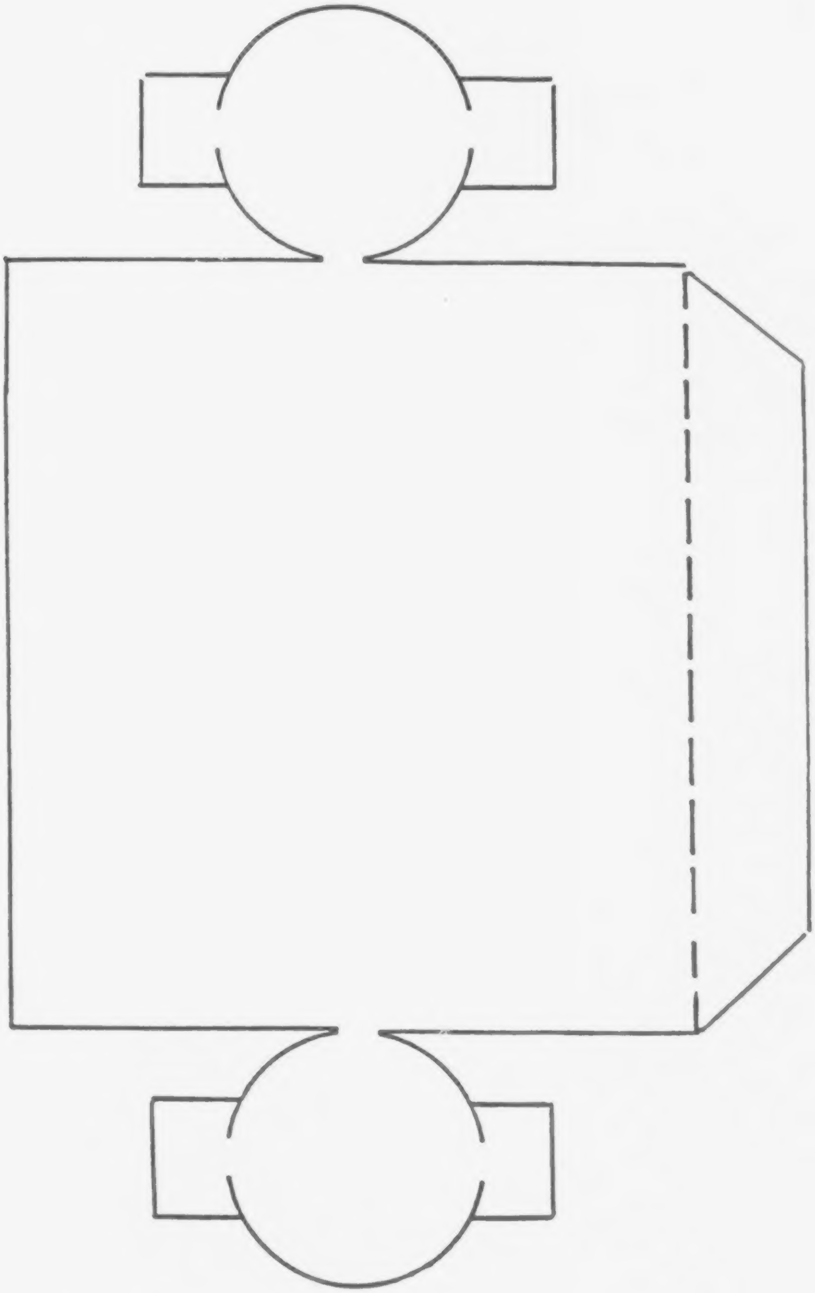
Form other regions similar to the following and devise methods of finding their areas.





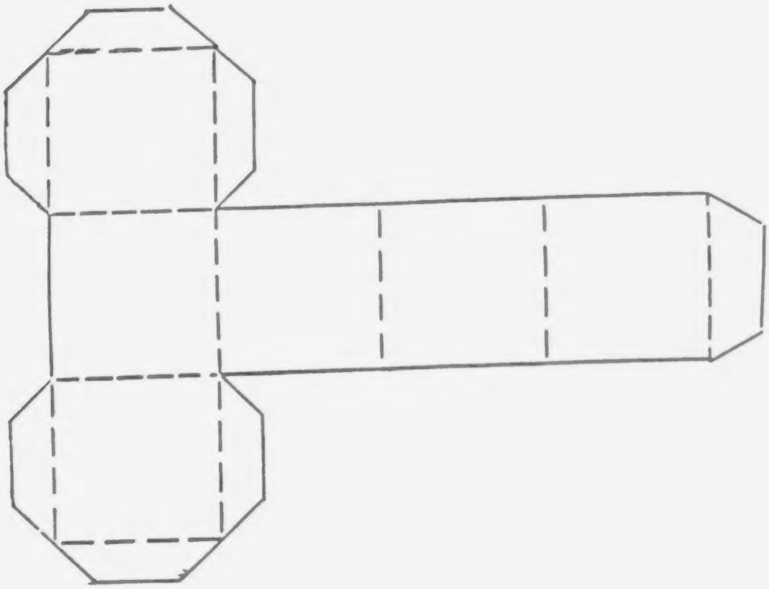
RESOURCE 10: THREE-DIMENSIONAL PATTERNS

CYLINDER

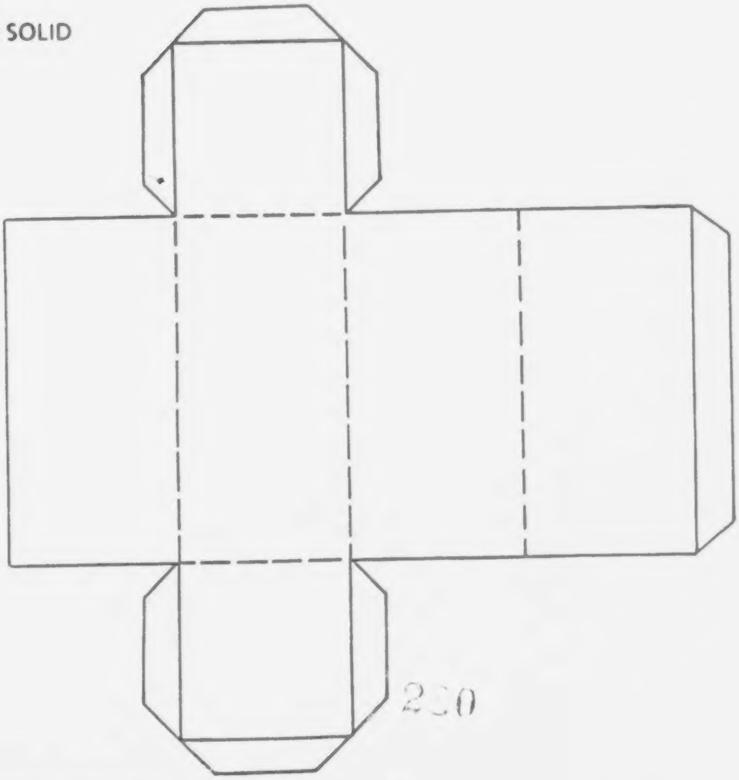


RESOURCE 10: THREE-DIMENSIONAL PATTERNS (continued)

CUBE



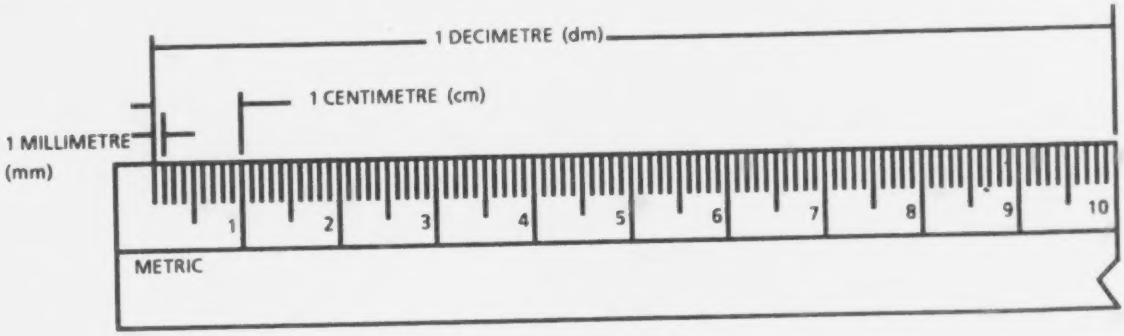
RECTANGULAR SOLID



RESOURCE 11: METERS, SCALES AND GAUGES

USING A METRIC RULER

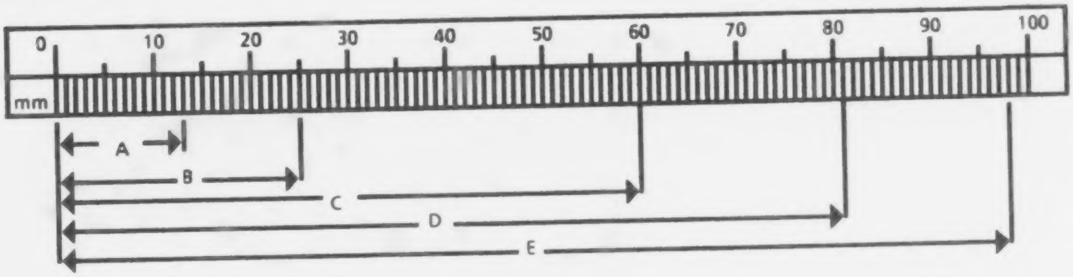
Study the scale on the metric ruler illustrated below.


1 DECIMETRE (dm)
1 CENTIMETRE (cm)
1 MILLIMETRE
(mm)
1
2
3
4
5
6
7
8
9
10
METRIC

METRIC SCALE

Exercises:

1. Using the two scales that follow, determine the length of each line to the nearest millimetre. Record your answer first in millimetres, then in centimetres.


0
10
20
30
40
50
60
70
80
90
100
mm
A
B
C
D
E

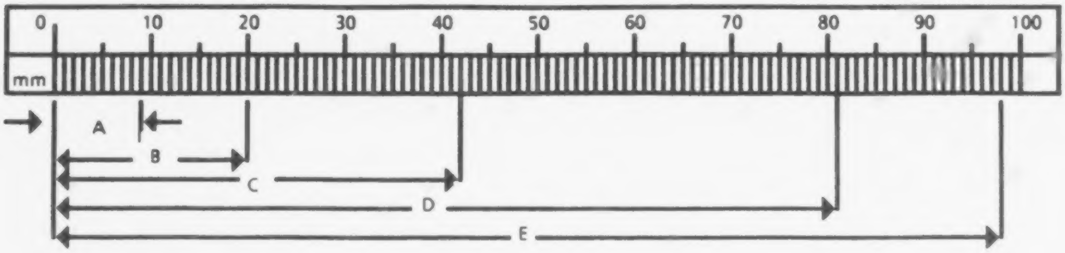
A ____ ____

B ____ ____

C ____ ____

D ____ ____

E ____ ____



A ____ ____

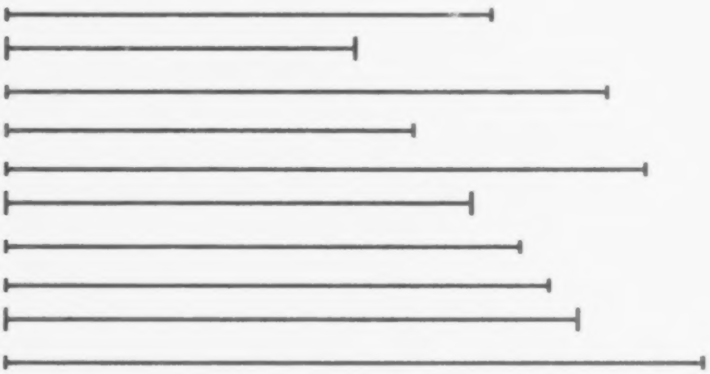
B ____ ____

C ____ ____

D ____ ____

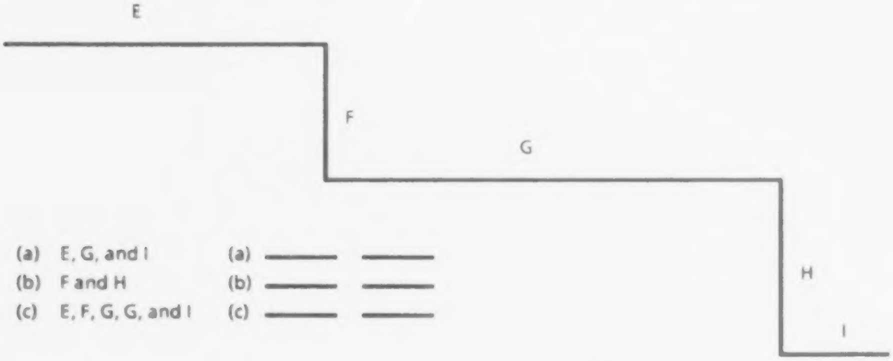
E ____ ____

2. Use a metric scale to measure the length of each line to the nearest millimetre. Record your answer first in millimetres, then in centimetres.



____ ____
____ ____
____ ____
____ ____
____ ____
____ ____
____ ____
____ ____
____ ____
____ ____

3. Use a metric scale to measure the total length of the following diagram. Record your answer first in millimetres, then in centimetres.

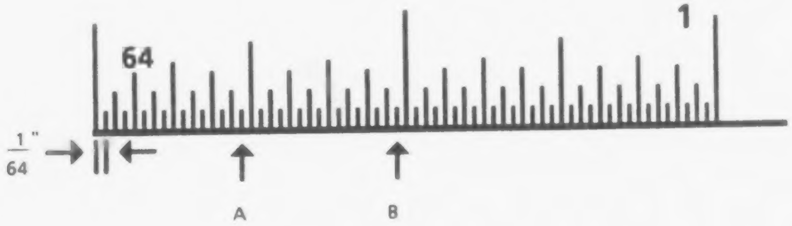


(a) E, G, and I (a) ____ ____
(b) F and H (b) ____ ____
(c) E, F, G, G, and I (c) ____ ____

USING AN INCH RULER

The diagrams below will help you read the different divisions on a ruler. Notice the number 64 printed on the ruler. This number means that the inch is divided into 64 parts, each part equal to 1/64th of an inch. Also notice that the divisions have lines of different length.

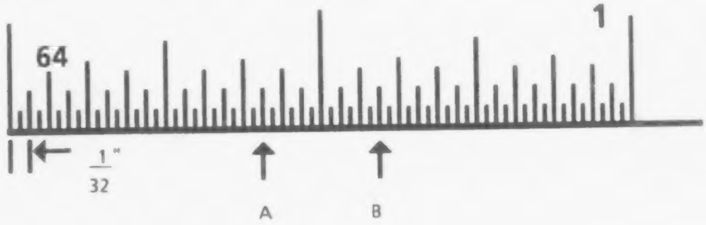
(a) The shortest line represents 64ths of an inch.



Point A represents: counting from the end, we count 15 divisions. Therefore, point A represents 15/64th's of an inch.

Point B represents: ______________________________.

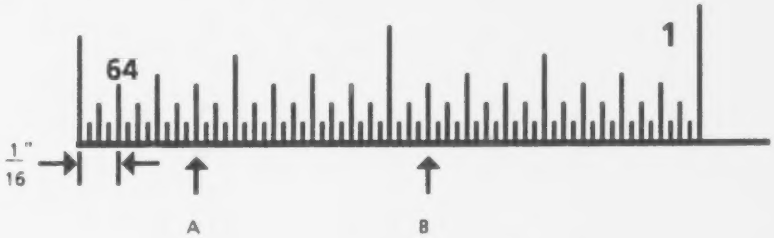
(b) The next longest line represents two parts of an inch (i.e., 1/64 + 1/64 = 2/64 = 1/32). Therefore, each of these lines represent 1/32nd of an inch.



Point A represents: counting every line of this length as well as any longer line, we count 13. Therefore, point A represents 13/32nd's of an inch.

Point B represents: ______________________________.

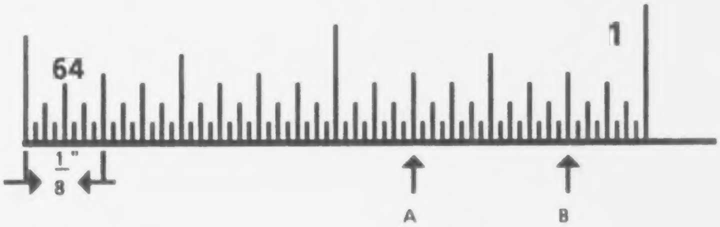
(c) The next longest line represents 4 parts of an inch (i.e., 1/64 + 1/64 + 1/64 + 1/64 = 4/64 or 1/16). Therefore, each of these lines represent 1/16th of an inch.



Point A represents: counting every line of this length or longer, we count 3. Therefore, point A represents 3/16th's of an inch.

Point B represents: ______________________.

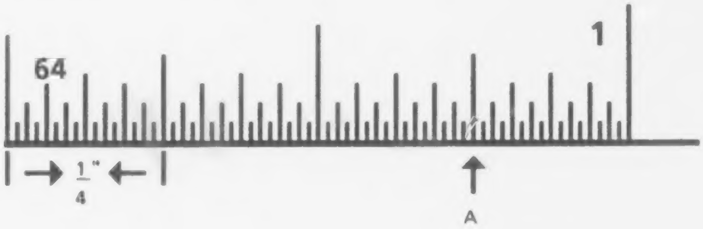
(d) The next longest line represents 8 parts of an inch (i.e., 8/64 or 1/8). Therefore, each of these lines represents 1/8th of an inch.



Point A represents: counting every line of this length or longer, we count 5. Therefore, point A represents 5/8th's of an inch.

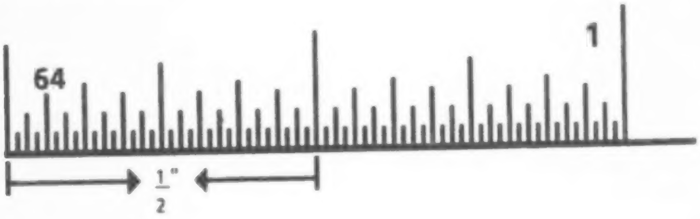
Point B represents: ______________________.

(e) The next longest line represents 16 parts of an inch (i.e., 16/64 or 1/4). Therefore, each of these lines represent 1/4 of an inch.

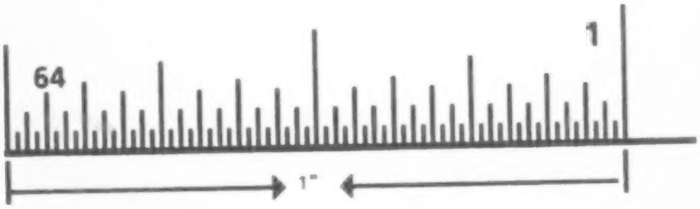


Point A represents: ______________________.

(f) The second longest line represents 32 parts of the inch (i.e., 32/64 or 1/2). Therefore, each of these lines represent 1/2 of an inch.



(g) The longest line represents one whole inch.

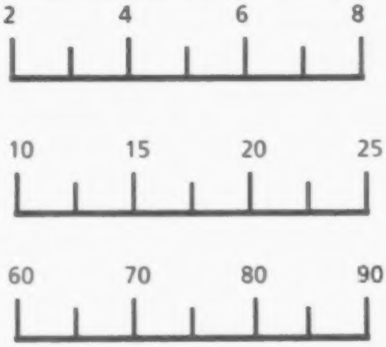


READING OTHER SCALES AND GAUGES

Before you read any scale, you must examine it carefully to see how it is calibrated.

- Look at the numbers written on the scale. They may not be sequenced in the usual order (e.g., 1, 2, 3, 4 and so on).

  Study the examples:



Numbers go up by 2's.

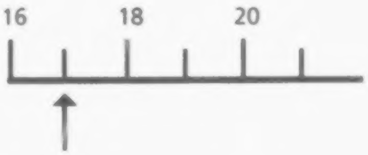
Numbers go up by 5's.

Numbers go up by 10's.

Other scales may have numbers that go up by 20's, 50's, 100's, or 1000's.

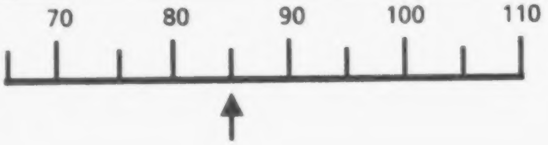
- Look at the marks between the numbers. The numbers represented by these marks depend upon:
  - the number of spaces between two consecutive numbers
  - the difference between two consecutive numbers on the scale.

  Study the examples:

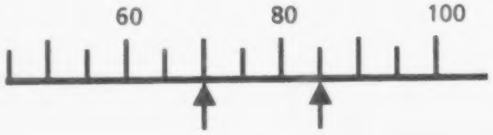


Numbers go up by 2's.
The arrow points to 17.

RESOURCE 11: METERS, SCALES AND GAUGES (continued)



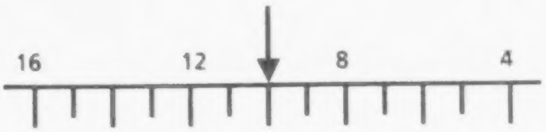
Numbers go up by 10's.
The arrow points to 85.



Numbers go up by 20's.
The first arrow points to 70.
The second arrow points to 85.

READING SCALES THAT GO BACKWARD

Sometimes a scale appears to go backward. These scales must be read from right to left, as in the example below.



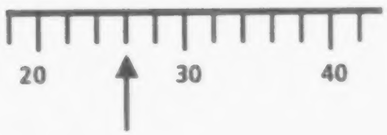
The reading is 10.

"Backward" scales are often found on meters that are turned by gears (e.g., a water meter or an electricity meter in a house).

READING MORE DIFFICULT SCALES

An easy way to figure out the value of each space on a scale is to divide the difference between two numbers on the scale by the number of spaces between them.

Example 1:



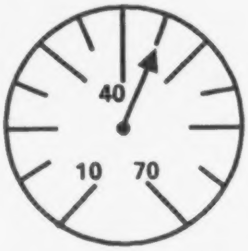
The difference between 20 and 30 is 10.

There are 5 spaces between 20 and 30.

Therefore, each space represents $\frac{10}{5}$ or 2 units.

Each space represents 2. The arrow indicates 26.

Example 2:



The difference between 40 and 70 is 30.

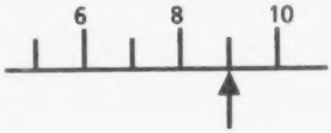
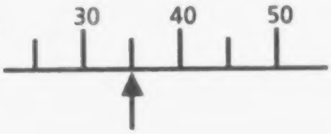
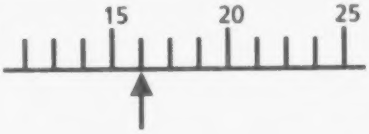
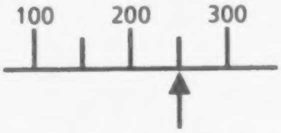
There are 6 spaces between 40 and 70.

Therefore, each space represents $\frac{30}{6}$ or 5 units.

Each space represents 5. The arrow indicates 45.

EXERCISES

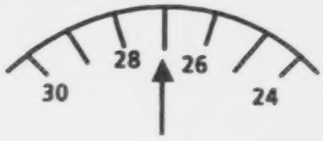
1. Look carefully at each scale. Then answer the questions.

(a) 	a. The numbers written on the scale go up by ____. b. The arrow points to the mark that means ___.
(b) 	a. The numbers written on the scale go up by ____. b. The arrow points to the mark that means ___.
(c) 	a. The numbers written on the scale go up by ____. b. The arrow points to the mark that means ___.
(d) 	a. The numbers written on the scale go up by ____. b. The arrow points to the mark that means ___.

RESOURCE 11: METERS, SCALES AND GAUGES (continued)

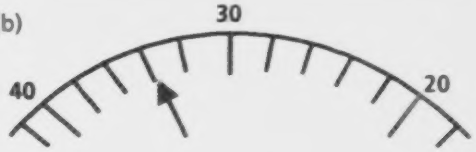
2. Read each of the following scales.

(a)



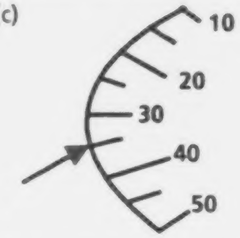
____

(b)



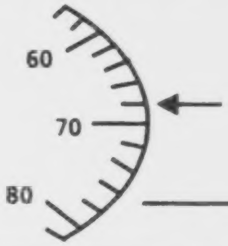
____

(c)


10
20
30
40
50

____

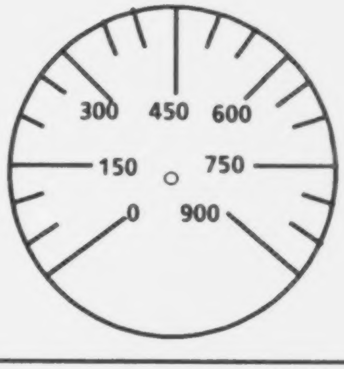
(d)



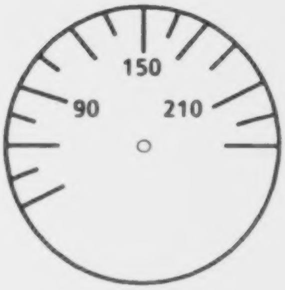
____

3. Give the value of each space on the scales illustrated below.

(a) ____________


300
450
600
150
750
0
900

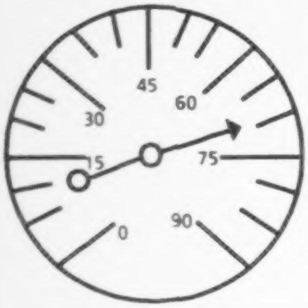
(b) ____________



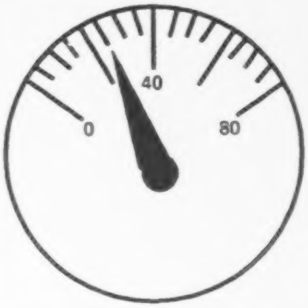
289

4. Give the reading indicated on each of the following scales.

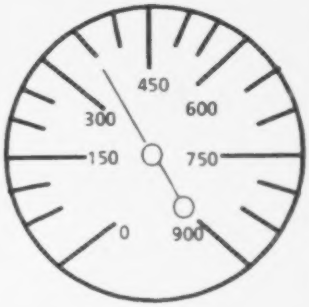
(a) ________


0
15
30
45
60
75
90

(b) ________

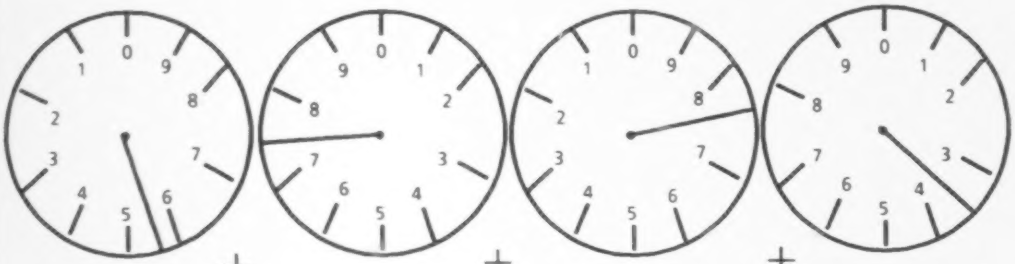


(c) ________


0
150
300
450
600
750
900

5. Give the readings indicated on each of the following electric meters.

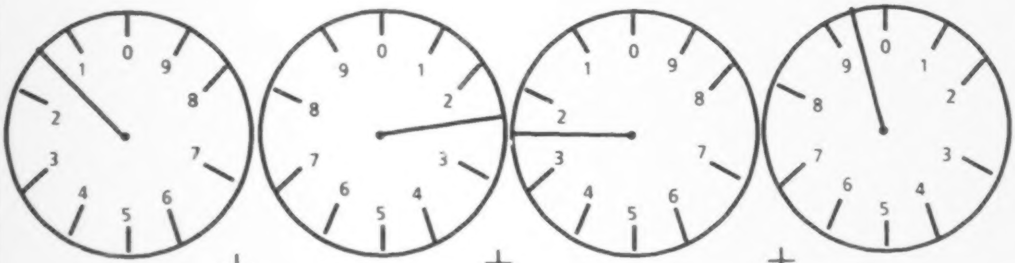
(a)


0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9

________ + ________ + ________ + ________

= ____kwh.

(b)


0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9

________ + ________ + ________ + ________

= ____kwh.

ASSESSMENT/EVALUATION

The mathematics program emphasizes an understanding of concepts/skills, problem solving and application. Assessment of understanding and problem solving must go beyond determining the percent of correct responses on a test based on mathematical skills. It should also provide information about how students approach doing mathematics, the level of understanding students have of concepts, and their ability to apply mathematics in new situations. The role of assessment is to provide feedback and evidence of progress toward all instructional goals. A singular assessment technique cannot provide this evidence.

In a report of the National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, the following statement is made about assessment and evaluation within the mathematics program:

> "A common form of assessment is testing for the purpose of assigning grades. But assessment should be conceived of as a much broader and basic task than just testing and grading. Its basic purpose is to determine **what** and **how** students think about mathematics. Assessment should involve the biography of students' learning as well as the continual impact of the instructional program. Such assessment should provide the basis for improving the quality of instruction. Indeed, assessment has no raison d'être unless it is clear how assessment can and will improve instruction"[1]

It is important to recognize that through the process of evaluation we evaluate students' **performance** and not students themselves. This understanding helps one avoid permanently classifying a student as a good student or a poor student. Performance can and does change, and teachers should be aware of these changes.

Suggestions and ideas for assessing and evaluating students' performance have been included in this section of the manual, and focus attention on the use of:

- Observation
- Interviews
- Inventories/Checklists
- Anecdotal Records
- Written Assignments
  - Paper-and-Pencil Tests
  - Diagnostic Writing Assignments.

The evaluation strategies suggested in this manual are not intended to be discrete, and should be used in conjunction with other strategies. For example, a checklist may be used to document desirable problem-solving behaviour in the classroom, and also to guide discussion and evaluate performance in an interview. Furthermore, teachers are encouraged to extend the application of these evaluation strategies to situations beyond those that are discussed.

Additional examples of evaluation strategies are provided in:

- *Curriculum and Evaluation Standards for School Mathematics*
- *How to Evaluate Progress in Problem Solving*.

Both of these resources are available through the Learning Resources Distributing Centre and a description of the resources can be found in Appendix A of this document.

---

[1] Reproduced with permission from *Curriculum and Evaluation Standards for School Mathematics: Working Draft*. Copyright 1987 by the National Council of Teachers of Mathematics.

OBSERVATION

Teachers observe all the time. When observations are documented, their effectiveness as an evaluation strategy increases immensely. Documented observations often provide the raw data required for analysis and diagnosis, and provide the basis on which to make remediation or enrichment decisions.

Mathematics lessons usually have a component in which students work individually or in small groups on assignments and projects. At this time, teachers can observe students at work, look for specific behaviours or outcomes, ask questions and make suggestions. Elements of the learning process that might be monitored through observation include:

- understanding of concepts/skills
- method of attacking problems
- work habits
- level of independence with work
- interpersonal skills and social growth.

Documentation of behaviours that are observed may occur in the form of anecdotal records or checklists. File anecdotal records and checklists in a student folder where samples of daily work, project reports and other artifacts are also placed.

CLARIFICATION/EXAMPLE

Anecdotal Record

Name	Date	Situation	Behaviour	Comment
John	1/17	group lesson, developing meaning of fractions with graph paper	quick to help neighbouring students	understands basic fraction concepts
	1/25	computation game	unable to use mental arithmetic when multiplying by powers of 10	redevelop and practise strategies for multiplying by powers of 10

A sample checklist that might be used in observing student behaviours has been provided as Resource 1: Observational Checklist of Student Behaviours.

Sometimes audiotapes or videotapes can be used to provide records that can be analyzed in more detail at a later time. Photographs are also useful in providing a record of project work. In addition to providing the tangible kinds of things that parents like to see at conference time, these records of student performance enable the teacher to measure progress with more objectivity than simply through memory of what was done.

INTERVIEWS

A planned interview with a student or group of students is an effective technique for assessing knowledge, understanding, thinking style, attitude or personal interests. An interview removes the restriction of writing, and enables the teacher to delve more deeply into how a student goes about finding an answer or solving a problem. Although written responses on an assignment may indicate areas of concern, more information is often required before appropriate remediation can be provided. Remediation strategies that are based solely upon the analysis of written responses may in fact be inappropriate at times. Holding interviews with students can reveal both unsuspected weaknesses and surprising strengths in their mathematical thinking.

Some guidelines for conducting interviews include:

- Establish an atmosphere of acceptance. The student must feel comfortable enough to verbalize his or her ideas freely. By accepting the student's responses without judgement, but with encouragement to elaborate further, you are communicating not only respect for the student's thinking but also a curiosity to learn more. Each response, whether right or wrong, has the potential of providing information about the student's level of understanding.
- Ask probing questions. During the interview, ask questions and introduce materials that will cause the student to extend and apply concepts/skills to new areas. Rephrase questions using vocabulary familiar to the student, so as to clarify both your intent and the student's thinking. Although some of the questions you ask should be planned, others should be invented spontaneously in order to test your hunches about the student's thinking. Questions asked may take some of the following forms:
  - How did you . . .?
  - Why did you . . .?
  - How do you know that . . .?
  - Have you . . .?
  - How did you decide whether . . .?
  - Can you describe . . .?
  - Are you sure that . . .?
  - How do you feel about . . .?
- Pace the interview appropriately. By allowing an adequate pause following each question before repeating or rephrasing it, you are giving the student time to interpret the question and construct the response. On some tasks, the student may need more than a thirty-second pause, whereas on others, three seconds will suffice. Also, by allowing an adequate pause following the student's response, you are indirectly encouraging the child to elaborate on his or her initial response.
- Be prepared to coax and encourage the student to make a response. A frequent response given by the student may be "I don't know" or "I forgot". Sometimes an extended pause can coax out productive thinking. At other times, the teacher can encourage a response to questions about which the student is unsure by saying:
  - "I know it's not easy to think about. Just give me your best idea."
  - "Pretend that you did know. How do you think it might be done?"

Interviews should have a definite purpose and both teacher and student should be aware of the purpose. Interviews must be planned in advance. In preparing for an interview, the teacher should consider:

- What questions will I ask?
- What basic understandings will I initially assess?
- How can I incorporate the use of manipulative materials?
- How can I vary the task and questions to obtain different perspectives on the student's ability?

Maximum benefits can be gained from an interview by reflecting on your interaction after listening to an audiotape playback. Considerable value can also be gained from sharing your tape with a colleague. By discovering how students interpret and view a problem, the teacher will be better able to make effective on-the-spot decisions in the classroom.

Several guides for interviewing have been included at the end of this section of the manual:

- Resource 2: Sample Protocol for Student Interviews
- Resource 3: Interview Guide for Problem Solving
- Resource 4: Interview Guide for Project Work

INVENTORIES/CHECKLISTS

Inventories and checklists are documentation strategies that are used conjunctively with other evaluation strategies. They can be easily designed and customized to meet many different needs and situations. Generally a matrix is created, with indicators of desirable behaviours/outcomes on one side, and ratings/skill levels along another side. As teachers note a particular behaviour, they need only check the appropriate column that evaluates or rates the behaviour.

Checklists lend themselves very well to documenting such elements of the program as:

- understanding of a concept when using a manipulative
- mastered knowledge, skills or process
- work habits
- problem-solving strategies
- social skills.

CLARIFICATION/EXAMPLE

Checklist/Inventory of Social Skills

Behaviour to be Observed	**Frequently**	**Sometimes**	**Never**
Is sensitive to the needs and problems of others.			
Willingly shares ideas/materials.			
Accepts suggestions and help.			
Adheres to group plans/decisions.			
Works cooperatively with others.			
Respects the property of others.			
Appears to like group work.			

A variety of inventories/checklists useful in monitoring the development of problem-solving skills are provided in the "Problem Solving" section of this manual:

- Attitude Inventory Items
- Observational Checklist of Problem-Solving Attitudes and Behaviours
- Observational Rating Scale of Problem-Solving Attitudes and Behaviours
- Checklist of Problem-Solving Strategies
- Problem-Solving Strategy Inventory.

Similar checklists can be developed to monitor and evaluate other components of the mathematics program.

e.g., – use of estimation strategies
– facility with mental arithmetic.

ANECDOTAL RECORDS

Anecdotal records refer to the spontaneous documentation of notable behaviour, effort and achievement. These records provide specific and dated information that can form the basis for conclusions and assessments. Anecdotal records often prove invaluable in clarifying assessments and add credibility to observations and recommendations being offered in student, parent and/or teacher meetings.

Anecdotal records may include observations on:

- attitude/work habits
- social skills
- effort and homework
- changes in performance
- specific strengths/deficiencies
- communication skills.

CLARIFICATION/EXAMPLE

Anecdotal Record Card

Student: Sue Jones
Date: 04/10

Comments:

- knows how and when to look for a pattern in problem solving
- keeps trying even when she has trouble finding a solution
- shows interest in helping others
- often enters numbers into the calculator in the wrong order when dividing

Anecdotal records may be kept in a daily or weekly diary, in student files, in the marks record book, or in a common file of short dated notes.

WRITTEN ASSIGNMENTS

PAPER-AND-PENCIL TESTS

Traditional paper-and-pencil tests are probably the most widely used method of evaluating student performance. While effective in assessing factual and procedural knowledge, these tests often elicit feelings of inadequacy and self-doubt for students who have experienced previous difficulty or failure. These negative feelings affect test performance, and may cause a cycle of repeated failure to continue.

Students may have learned the information presented in class, but are unable to demonstrate the knowledge because of poor reading skills, visual perception problems, inadequate reasoning and comprehension, fine-motor difficulties or other related deficiencies. Special needs of the student can be met through minor alterations in the construction of teacher-made tests. Constructing tests according to special needs can mean the difference between success and failure for some students.

The suggestions that follow will assist teachers to construct their tests according to the needs of individual students.

TEST DIRECTIONS

- Keep directions simple and avoid unnecessary words.
- Define words that are unfamiliar or abstract.
- Give an example of how the student is to respond.

CLARIFICATION/EXAMPLE

Directions

Add the fractions.
Give the answer in lowest terms.

Example:

$$\begin{array}{rr} & \frac{1}{3} = \frac{2}{6} \\ + & \\ & \frac{1}{2} = \frac{3}{6} \\ \hline & \frac{5}{6} \end{array}$$

Some teachers feel that providing a model invalidates a test designed to measure knowledge of mathematical process. However, some students may never be able to remember a formula or complex set of processes without visual prompts. For them, failure is almost a certainty without modifications.

- Avoid oral directions as the only means of communication. Read directions orally as well as clearly writing them on the test.

TEST ITEMS

- Provide manipulative objects that make the problems more concrete.
- When using computation problems, avoid mixing different problem formats in the same section.

CLARIFICATION/EXAMPLE

A student with organizational or visual tracking problems may have difficulty in "changing gears" from problem A to problem B. It might be better to put these questions in two different sections of the test.

Problem A

$$\begin{array}{r} 468 \\ +\ \ 31 \\ 896 \\ \hline \end{array}$$

Problem B

$$670 + 40 + 861 =$$

- Provide visual prompts for computational problems.

CLARIFICATION/EXAMPLE

$$\begin{array}{r} \square\ \ 73 \\ +\ \ 69 \\ \hline \end{array} \qquad \text{OR} \qquad \begin{array}{r} \text{R} \\ 61\overline{)263} \\ -\ \underline{\qquad} \end{array}$$

- Provide formulas and explain the meaning of special symbols (e.g., $A = 1 \times w$, $<$ means "less than").
- Provide a set of written steps for applying difficult algorithms.

CLARIFICATION/EXAMPLE

Long Division

1. Divide
2. Multiply
3. Subtract
4. Check
5. Bring down

- Incorporate the use of the calculator into computational problems that are not intended to assess understanding of the paper-and-pencil process.
- Design word problems that:
  - are relevant to the student's personal experience
  - consist of simple sentences and familiar words.
- Underline or circle key words in word problems (e.g., less, more).
- Ask students to circle the correct response in multiple choice items. This reduces the possibility of copying errors when transferring letters to blanks. Arrange the answer and distractors vertically on the page.

CLARIFICATION/EXAMPLE

You have a board 48 centimetres long. If you cut off a 6-centimetre piece, how much is left?

a. 38 centimetres
[✓ b. 42 centimetres]
c. 48 centimetres

- Keep all matching items brief, and have only one correct answer for each item. Use no more than ten items in the matching lists. If you have more than ten items, group them by concepts in clusters of ten.

TEST DESIGN

- Construct the test in logical sequential order, from simple to complex problems.
- Use test items that reflect the content taught and techniques used to teach.
- Prepare a study guide for the test that matches the design of the actual test.
- Design the test to reflect the student's knowledge, rather than ability to follow complicated directions, to use difficult vocabulary, or to work under time constraints.
- Adjust the readability level of the test to meet student needs.
- Prepare the test in short sections that can be administered individually if necessary.
- Use graph paper for paper-and-pencil computational problems. The squares may help the student to keep figures aligned.

DIAGNOSTIC WRITING ASSIGNMENTS

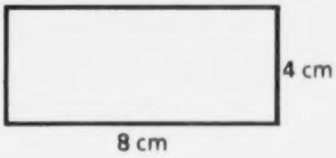
Communication, both oral and written, plays a vital role in the teaching and learning of mathematics. Students should not only be involved in group problem-solving activities where they must share ideas with one another, but should also have the opportunity to express their ideas in writing.

Writing in mathematics requires students to reflect on their reasoning and thus enhances cognitive development. Diagnostic writing assignments require students to respond to specific mathematical questions in an expressive writing style. Written responses often force students to examine their own understanding of concepts, and communicates to teachers how much students really know about a concept. Written responses also provide insight into how students think. Information revealed through written responses will enable teachers to diagnose students' strengths and weaknesses, and plan lessons that promote mathematical thinking and understanding. Appropriate remedial and enrichment activities can be determined on the basis of understanding demonstrated for a given concept.

CLARIFICATION/EXAMPLE

Comparing Perimeter and Area

What is wrong with each of the following statements about the rectangle?



- The area is 32 cm.
- The perimeter is 12 cm.
- The area is 12 cm^2.
- If the length is increased by 2 cm, the area will be increased by 2 cm^2.
- If the perimeter is doubled, the area will be doubled.
- The perimeter of any rectangle is always smaller than the area.

Diagnostic writing assignments (i.e., writing to describe thinking and reasoning) should be kept relatively short. As this type of writing may be new to your class, encourage students to spend time going beyond the "right answer". Written assignments should not be graded for a mark but, rather, assessed for understanding. Students might be awarded bonus points based on their effort and presentation.

CLARIFICATION/EXAMPLE

0 points	no effort
1 point	mediocre attempt with little or no understanding
2 points	good attempt, but with some lack of understanding
3 points	high level of effort and understanding

Diagnostic writing assignments can be kept in a student diary or logbook, and might be assigned on a regular basis (once or twice a week), or in lieu of a regular quiz.

RESOURCE I: OBSERVATIONAL CHECKLIST OF STUDENT BEHAVIOURS

LEVEL OF INDEPENDENCE WITH WORK:

	ALWAYS	OFTEN	SOMETIMES	NEVER
• Settles down to work upon entering class.				
• Spends time on task; has satisfactory attention span.				
• Takes responsibility for making up work after absences.				
• Takes responsibility for supplies and equipment.				
• Follows directions; completes tasks with minimal assistance.				
• Asks for and accepts help when needed.				
• Accepts a challenge; works productively on tasks of increasing difficulty.				
• Displays self-confidence and pride in work.				

UNDERSTANDING OF CONCEPTS AND SKILLS:

• Uses the necessary vocabulary and concepts.				
• Uses appropriate operations, strategies and principles.				
• Asks questions, volunteers answers.				
• Answers questions that involve thought (e.g., What do you think?).				
• Demonstrates understanding through ability to generalize and apply.				
• Displays curiosity about concepts, relationships and applications.				
• Works independently on projects and research.				

RESOURCE I: OBSERVATIONAL CHECKLIST OF STUDENT BEHAVIOURS (continued)

ABILITY TO INVESTIGATE AND SOLVE PROBLEMS:

	ALWAYS	OFTEN	SOMETIMES	NEVER
• Understands and defines problems.				
• Develops a systematic plan of attack.				
• Gathers information using a variety of sources.				
• Carries out plans and procedures, seeking help when necessary.				
• Uses appropriate strategies and processes.				
• Considers alternatives before reaching a solution/decision.				
• Evaluates solutions to the problem and decisions made.				
• Considers other ideas/opinions/solutions.				

INTERPERSONAL SKILLS AND SOCIAL GROWTH :

• Resists aggressive and impulsive behaviours.				
• Volunteers to work in group situation.				
• Cooperates and contributes to group goals.				
• Listens to peers; considers the opinions of others.				
• Participates in oral discussions.				
• Helps others willingly.				

RESOURCE 2: SAMPLE PROTOCOL FOR STUDENT INTERVIEWS[1]

Sample interview questions that might be used in assessing the student's understanding of operations, basic facts and algorithmic procedures are provided. It is not suggested that every question be used during each interview. Their purpose is simply to illustrate how carefully sequenced and well-phrased questions can provide worthwhile information about the student's understanding of process and skill.

UNDERSTANDING OF OPERATIONS

The following questions and tasks relate to the operation of division. Similar questions and tasks could be developed for other operations. These questions will indicate:

- how students interpret the operation
- whether the action can be identified and simulated
- whether the operation can be related to personal experience.

INTERVIEW QUESTIONS:

1. Read "56 ÷ 7" for me.

2. Do you know other ways of reading this? Students may frequently read statements like 56 ÷ 7 or equivalent subtraction statements (e.g., 56 − 7) in either direction.

3. Do you know other names for " ÷ "? (Point to the symbol, or use terminology from the student's response to the previous task.)

4. Use counters (or draw a sketch) to show 12 ÷ 3.
   - Is the student able to simulate the action for 12 ÷ 3 with counters?
   - Is the student's preferred division interpretation one of measurement or partition, and will the student be consistent throughout the interview?

5. Make up a word problem for 12 ÷ 3.

KNOWLEDGE OF BASIC FACTS

The following examples deal with multiplication facts. Similar strategies could be developed for basic facts related to other operations. Through appropriate interview questions, the teacher can assess the student's ability to:

- recall simple basic facts
- use known facts to derive answers for other facts
- apply the properties of "zero" and "one"
- use strategies for checking answers or finding an answer for a fact that is not known.

---

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1988 by the National Council of Teachers of Mathematics.

INTERVIEW QUESTIONS:

1. Give the answer for 4×0.
2. What is your rule for finding the answer when a number is multiplied by zero? Use your rule for 25×0.
3. Give the answer for 6×1.
4. What is your rule for finding the answer when a number is multiplied by one? Use your rule for 37×1.
5. If the answer for 7×9 is 63, how could you use this answer to find the answer for 8×9?
6. Pretend you forgot the answer for 7×8. How would you find or calculate this answer?
7. As I show you a flash card, tell me, without calculating the answer, whether you think the fact is easy or hard. (After the cards have been sorted as "easy" and "hard", answers are solicited and checked by turning the appropriate cards over, first for the facts classified as easy and then for those identified as hard.)

UNDERSTANDING OF ALGORITHMIC PROCEDURES

Subtraction is used to illustrate a possible protocol for an algorithmic procedure during an interview. These procedures can be adapted for other operations. Questions posed during the interview should relate to:

- the order of "attack" used by the student
- the reasons for "moves" made during the calculation
- the meaning of the digits that are being manipulated.

INTERVIEW QUESTIONS:

1. Show me how you would find the answer for 57 – 34 = ____. Talk to me as you are doing it.

   Explanations provided by the student should provide answers for the following questions:
   - Why did you record one number below the other?
   - Why did you begin "here"?
   - You said "5 – 3". What do these digits really mean?

2. Explain to me what you are thinking as you find the answer for 71 – 48 = ____.
3. Show me how you would explain how to find the answer for 4003 – 897 = ____ to a student in a lower grade (or to a younger brother or sister).
4. Use these base ten blocks to show how you would find the answer for 605 – 67 = ____.

RESOURCE 3: INTERVIEW GUIDE FOR PROBLEM SOLVING[1]

1. Establish rapport to help the student feel comfortable.

2. Ask the student to "talk about what he or she is doing or thinking" while solving the problem. Explain that this will enable you to help the student to become a better problem solver.

3. Give this problem to the student:

   At an amusement park, Mike and his 5 friends decided to take enough roller coaster rides so that each person would take a ride with every other person exactly once. How many rides were taken if only 2 students went on each ride?

4. As the student attempts to understand the problem question and conditions, observe the student and ask questions such as the following, if appropriate:

   a. What did you do first when given the problem? Next?
   b. What question is asked in the problem? What are the important facts and conditions in the problem? Do you need any information not given in the problem?
   c. Is there anything you don't understand about the problem?

5. As the student works on a solution to the problem, remind him or her again to talk about it, and ask questions such as the following, if appropriate:

   a. What strategy are you using? Do you think it will lead to a solution? Have you thought about using other strategies? Which ones?
   b. Where are you having difficulty? What are your ideas about where to go from here?

6. As the student finds an answer to the problem, observe the ways, if any, in which he or she checks the answer and its reasonableness as a solution. Ask questions such as:

   a. Are you sure this is the correct answer to the problem? Why?
   b. Do you think it is important to check your answer? Why?

7. After the student has solved the problem, ask questions such as:

   a. Can you describe the solution to the problem and how you found it?
   b. Is this problem like any other problem you've solved? How?
   c. Do you think this problem could be solved in another way? What are your ideas?
   d. How did you feel while you were solving this problem? How do you feel now that you have found a solution?

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

RESOURCE 4: INTERVIEW GUIDE FOR PROJECT WORK[1]

THE TASK ENVIRONMENT

1. How did you select this investigation/project? ______________________________

____

2. Are you interested in it?
____ very _____ somewhat _____ not at all

3. What did the teacher do when giving out the assignment? ______________________

____

Examples:
- give verbal guidelines
- give written guidelines
- select the topic
- provide a strategy
- increase your interest

4. Who do you expect will examine the results of your investigation/project? ____________

____

PREVIOUS KNOWLEDGE

5. Have you undertaken an investigation/project like this before?
____ yes _____ no

6. What did you know about this topic before you started? ______________________

____

7. Which of the following mathematical ideas/processes did you use in carrying out your investigation/project?

_____ geometric shapes
_____ order of operations
_____ fractions
_____ rounding numbers
_____ mental arithmetic
_____ units of measure
_____ banking skills
_____ scale drawing

_____ large numbers
_____ decimals
_____ estimation
_____ calculator usage
_____ graphs and tables
_____ budgeting
_____ comparative shopping
_____ patterns and designs

---

[1] For permission to reprint copyrighted material grateful acknowledgement is made to the following: Division for Learning Disabilities, *Learning Disabilities Focus* for the material adapted from "Alternatives in the Assessment of the Learning Disabled Adolescent: A Learning Strategies Approach" by Judith Wiener, Spring 1986.

RESOURCE 4: INTERVIEW GUIDE FOR PROJECT WORK (continued)

PLANNING

8. How did you plan your investigation/project? ______________________________

____

9. Did you make an outline of the steps you would take?
____ yes _____ no

What kind of thinking did you do first? ______________________________

____

10. Did you do any research? ______ yes ______ no

What sources of information did you use? ______________________________

____

REVIEWING AND APPLYING

11. What strategies did you find most helpful in completing your investigation/project?

Examples:
_____ reading and research
_____ following directions received
_____ explanations from the teacher
_____ studying diagrams/models
_____ receiving help from parents
_____ studying with a partner
_____ explaining problems to classmates
_____ having group discussions about solutions
_____ other

12. Explain three ways that you might make use of the results of your investigation in everyday life

____

____

____

EVALUATING

13. What have you learned in completing this investigation/project? ______________________

____

14. What grade did you think you would get? ______________________________

Why? ____

15. What was the teacher's evaluation? ______________________________

How was this evaluation different from your anticipated grade? ______________________

____

APPENDIX A: ANNOTATED LIST OF LEARNING RESOURCES

STUDENT RESOURCES

A Student Guide to Banking: Students' Portfolio. Toronto, Ontario: Canadian Imperial Bank of Commerce. Provides specimen bank forms and a short history of banking. Available upon request from the Canadian Imperial Bank of Commerce.

Alberta Consumer and Corporate Affairs. *Consumer Complaints: A Self-Help Handbook*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1985. Provides information that will help students resolve consumer complaints about products that don't perform as they should, inadequate services, and businesses that fail to live up to the terms of their sales contracts. Information is presented in magazine format. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *"Consumer Talk" Information Sheets*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs. Consists of a series of eight consumer information sheets presented in tabloid-newspaper format. Provides information useful in developing awareness of consumer rights/responsibilities and personal strategies for dealing with consumer problems. Information sheets in the series include *Buying a Used Car*, *Credit*, *Shop Around*, *Shopping at Home*, *Returns*, *Tenants: Moving In*, *Tenants: Living There* and *Tenants: Moving Out*. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *Moving Out*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1986. Provides information that is useful in developing consumer skills. Topics relate to housing, credit, transportation, the marketplace, lifestyle and careers. Information is presented in magazine format. Class sets are available upon request. An *Instructor's Guide* is also available. The guide provides programming ideas, discussion questions, case studies, community resources and activity suggestions.

Alberta Consumer and Corporate Affairs. *Taking Charge of Your Money*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1986. Provides strategies for making the most of your money through personal planning and budgeting. Topics relate to setting financial goals, estimating costs, planning spending, keeping records and using credit. Information is presented in magazine format. Class sets are available upon request.

Cutting, Robert, et al. *Math You Need, Book 2*. Scarborough, Ontario: Nelson Canada, A Division of International Thomson Limited, 1982. A workbook of developmental and practice activities that involve the application of mathematical skills in everyday situations. Topics relate to mathematics in sports, automotive mathematics, keeping a place of your own, spending and earning money, savings and interest, loans and credits, and buying insurance. The workbook also provides a bank of exercises on basic computation and measurement skills that students need to be able to use in coping with everyday situations.

Dladay, Les. *Decisions, Making Personal Economic Choices*. St. Paul, Minnesota: EMC Publishing Corporation, 1985. A workbook that provides an extensive set of student activity worksheets on finding employment, computing income, using bank accounts, preparing budgets and comparison shopping. Activities focus attention on the use of critical and creative thinking skills when making personal economic choices. Some topics involve references to American standards/practices and do not reflect the Canadian situation.

Elrick, Thomas. *Forms in Your Life: A Student Workbook and Guide to Everyday Forms*. Toronto, Ontario: Globe/Modern Curriculum Press, 1981. A workbook that provides guidelines for using a variety of forms encountered in everyday and work-related situations. Specimen forms included in the publication are those currently used by Canadian companies, institutions and federal and provincial government agencies.

Lennox, Art. *Banking, Budgeting and Employment*. Phoenix, New York: Frank E. Richards Publishing Company Incorporated, 1984. A workbook that is designed to supplement instruction in common banking, budgeting and employment terminology and procedure. The publication includes crossword and maze puzzles, vocabulary lists, exercises and story problems. Material within the workbook has been simplified and is very readable. Some activities involve references to American standards/practices and do not reflect the Canadian situation.

Lyng, Merwin, et al. *Applied Mathematics*. Markham, Ontario: Houghton Mifflin Canada Limited, 1989. A student textbook that supports the development of mathematical skills frequently used in the world of work. Activities focus attention on industrial measurements, algebra in the workplace, applied geometry, occupational simulations, and trigonometry and statistics at work. A *Teacher's Manual and Answer Key* that provides teaching strategies, warm-up activities, unit tests, teacher aids and a complete answer key is also available from the publisher.

Wool, John. *Useful Arithmetic, Volume 1*. Phoenix, New York: Richards Publishing Company, 1987. A workbook designed to assist students to apply mathematical skills in practical situations. Activities focus attention on buying goods and services, comparing prices, the work week, pay deductions and the cost of different transportation alternatives. Material within the workbook has been simplified and is very readable. Some activities involve references to American standards/practices and do not reflect the Canadian situation.

Wool, John. *Useful Arithmetic, Volume 2*. Phoenix, New York: Richards Publishing Company, 1988. A workbook designed to assist students to apply mathematical skills in practical situations. Activities focus attention on money problems, mathematics in the marketplace, earning money and banking services. Material within the workbook has been simplified and is very readable. Some activities involve references to American standards/practices and do not reflect the Canadian situation.

TEACHER RESOURCES

Alberta Education. *Problem Solving Challenge for Mathematics*. Edmonton, Alberta: Alberta Education, 1985. Produced as a junior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, a bank of problems, methods of evaluating problem solving in performance and a 36-item problem-solving biography.

Alberta Education. *Problem Solving in Mathematics: Focus for the Future*. Edmonton, Alberta: Alberta Education, 1987. Produced as a senior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, project and computer related problems, a bank of mathematics problems, methods of evaluating problem solving performance and a 123-item problem-solving bibliography.

Charles, R., et al. *How to Evaluate Progress in Problem Solving*. Reston, Virginia: The National Council of Teachers of Mathematics, 1987. Discusses goals of problem-solving evaluation, evaluation techniques, organizing and managing an evaluation program, and using evaluation results.

Easterday, K., et al. *Activities for Junior High School and Middle School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1981. Provides meaningful activities in a wide range of mathematics topics (e.g., a manipulative approach to integer addition/subtraction; paper-folding activities for exploring two-dimensional relationships; games that provide practice in mental arithmetic/fractions/percent/decimals/proportions/measurement/geometry; probability activities using non-standard dice/marbles/the alphabet; a card game to practise fraction-decimal-percent equivalencies; alternative approaches for developing equation-solving skills).

Hope, J., et al. *Mental Math in Junior High*. Palo Alto, California: Dale Seymour Publications, 1988. Provides teaching notes and blackline masters for a series of 50 lessons in mental calculation. Topics covered include working with compatible numbers, working from the front-end, dealing with trailing zeros, as well as more advanced lessons that develop methods of calculating mentally with fractions, decimals and percents. The publisher grants permission to individual teachers who have purchased this book to reproduce the blackline masters as needed for use with their own students.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1989. Provides a set of standards for mathematics curricula in North American schools (K-12) and for evaluating the quality of both the curriculum and student achievement. The document represents the consensus of NCTM's members about the fundamental content that should be included in the school mathematics curriculum and about key issues concerning the organization and implementation of student and evaluation programs. Establishes a broad framework to guide reform in school mathematics in the next decade.

TECHNOLOGY AND MEDIA

COMPUTER SOFTWARE

Managing Lifestyles: Survival Math Skills. Pleasantville, New York: Sunburst Communications Incorporated, 1984. (Apple II, TRS-80.) This package provides three simulations that are modelled after practical situations that students may encounter in everyday life. ("Comparison Shopping" involves shopping for a week's supply of groceries and planning a menu. "Budget for Success" requires students to plan and manage living expenses for six simulated months. "Calorie Counter" incorporates students' personal data into an activity involving counting calories to help maintain a desired weight.) Simulations within the package are designed to enhance students' ability to perform basic operations with whole numbers/decimals/fractions/percent, to develop and use formulas, and to determine unit prices and interest rates. The package includes one disk, backup, teacher's guide with reproducible worksheets, masters for overhead transparencies and student activities.

APPENDIX A: ANNOTATED LIST OF LEARNING RESOURCES (continued)

Math Strategies: Problem Solving. Willowdale, Ontario: Science Research Associates (Canada) Limited, 1983. (Apple II, TRS-80.) This program is designed to provide instruction and practice in solving multiple-step problems using appropriate strategies. Topics include simplifying a problem, breaking a problem into parts, identifying needed additional information, and making a model of the problem. The package includes two disks, a teacher's guide and 20 student texts.

Money Manager. Washington, D.C.: Computer Age Education Incorporated, 1984. (Apple II.) This program is a simulation designed to help students develop and practise skills in financial management. Students begin with a set amount of money in cash and in their chequing and savings account. During the simulated eight week period, the user earns money, pays bills, makes purchases, manages cash, controls bank accounts, and makes other financial decisions. The package includes one disk, a teacher's guide and a user's guide.

SRA Computer Drill and Instruction, Mathematics, Level D. Willowdale, Ontario: Science Research Associates (Canada) Limited, 1981-83. (Apple II, IBM PC.) This program provides drill exercises in computation, numbers and numeration, fractions, decimals, ratio and percent, measurement, pre-algebra and applications. The program does not provide answers or a method of solving problems. May be used by students in unmanaged or teacher-managed form. Multi-disk package includes a teacher's guide.

Spreadsheets for Students. Mississauga, Ontario: Copp Clark Pitman Limited, 1985. (Apple II, Commodore 64.) This spreadsheet-style computer program requires students to apply mathematical skills to practical situations involving shopping and sales tax, simple budgeting, savings accounts and loan variables. Activities within the program encourage students to estimate, approximate and round, formulate strategies and predict outcomes. The software is simplified and does not present full-size spreadsheet programs with advanced features. The package includes two disks, a set of reproducible blackline masters, and teacher's notes.

VIDEO PROGRAMS

Mathways. Kinetic, 1980. Program consists of a series of four 15-minute videos on various topics in mathematics. Designed to develop/review concepts and skills for students in Grades 7 to 12. Method of presentation is animation with voice-over. Available through ACCESS NETWORK and/or Regional Resource Libraries.

> *The Decimal Point*: Introduces or reviews fundamental decimal point concepts. Explains the meanings of various positions of the decimal point within a numeral.
> *The Percent*: Introduces the percent symbol. Uses a "sales pitch" to convince the viewer that the percent symbol is the hardiest symbol ever devised.
> *Areas*: Assumes that prior to viewing this program students understand the general concept of area and how to find areas of triangles and quadrilaterals. Introduces the symbol π and its use in computing area of a circle and surface area of a cylinder.
> *Volumes*: Uses layers of cubes to fill a rectangular prism to illustrate volume. Covers the computation of the volume of prisms, cylinders and cones.

APPENDIX A: ANNOTATED LIST OF LEARNING RESOURCES (continued)

Math Wise. Bloomington, Indiana: Agency for Instructional Television, 1981. This part-documentary and part-drama series consists of twelve 15-minute videos. The program is designed to motivate students to use math skills in making better decisions and solving problems in everyday situations. The videos are not mathematics "lessons"; rather, the actors use mathematics in a variety of practical situations. A commentator introduces the mathematics topic of each video and closes by reviewing the mathematically significant points in each of the "stories". The program provides a teacher's guide that suggests ideas for discussion and a student worksheet for each of the videos. Available through ACCESS NETWORK and/or Regional Resource Libraries.

Measuring 01: *Measuring Instruments*
Measuring 02: *Formulas*
Comparing 01: *Numerical Comparisons*
Comparing 02: *Percent*
Comparing 03: *Finding a Common Unit*
Comparing 04: *Proportion*
Locating/Interpreting 01: *Organizing Information*
Locating/Interpreting 02: *Graphs*
Locating/Interpreting 03: *Maps, Charts, and Tables*
Predicting 01: *Probability*
Predicting 02: *Choosing a Sample*
Predicting 03: *Averages*

Street Cents. The Canadian Bankers' Association, 1989. This program consists of twelve 30-minute videos designed to develop prudent judgement and consumer skills among Canadian teens. The program features food reports, product surveys, views from the marketing and advertising worlds, and suggests ways for teens to save and make the most of their money. Available through both ACCESS NETWORK and regional offices of Consumer and Corporate Affairs.

Program 1: *Advertising*
Program 2: *Food*
Program 3: *Jobs*
Program 4: *Music*
Program 5: *Fashion*
Program 6: *Future*
Program 7: *Millionaire*
Program 8: *The Environment*
Program 9: *Cheap Stuff*
Program 10: *Image*
Program 11: *Holidays*
Program 12: *Wrap-Up*

APPENDIX B: ALBERTA CONSUMER AND CORPORATE AFFAIRS OFFICES

Calgary

310 Centre 70
7015 Macleod Trail South
Box 5880, Postal Station "A"
Calgary, Alberta
T2H 2M9
Phone: 297-5700

Edmonton

3rd Floor Capilano Centre
9945 - 50 Street
Edmonton, Alberta
T6A 0L4
Phone: 427-5782

Fort McMurray

Fort McMurray Provincial Building
5th Floor West Tower
9915 Franklin Avenue
Fort McMurray, Alberta
T9H 2K4
Phone: 743-7231

Grande Prairie

Grande Prairie Provincial Building
10320 - 99 Street
Box 7
Grande Prairie, Alberta
T8V 6J4
Phone: 538-5400

Lethbridge

300 Professional Building
740 - 4 Avenue South
Bag Service 3014
Lethbridge, Alberta
T1J 4C7
Phone: 381-5360

Medicine Hat

303 Provincial Building
770 - 6 Street S.W.
Medicine Hat, Alberta
T1A 4J6
Phone: 529-3535

Peace River

Peace River Provincial Building
9621 - 96 Avenue
Bag 900, Box 9
Peace River, Alberta
T0H 2X0
Phone: 624-6214

Red Deer

2nd Floor Provincial Building
4920 - 51 Street
Red Deer, Alberta
T4N 6K8
Phone: 340-5241

APPENDIX C: REGIONAL OFFICES OF EDUCATION

Grande Prairie Regional Office

12th Floor, 214 Place
9909 - 102 Street
Grande Prairie, Alberta
T8V 2V4
Phone: 538-5130
FAX: 538-5135

Edmonton Regional Office

7th Floor, Westcor Building
12323 Stony Plain Road
Edmonton, Alberta
T5N 3Y5
Phone: 427-2952
FAX: 422-9682

Red Deer Regional Office

3rd Floor West, Provincial Building
4920 - 51 Street
Red Deer, Alberta
T4N 6K8
Phone: 340-5262
FAX: 340-5305

Calgary Regional Office

Room 1200, Rocky Mountain Plaza
615 Macleod Trail, S.E.
Calgary, Alberta
T2G 4T8
Phone: 297-6353
FAX: 297-3842

Lethbridge Regional Office

Provincial Building
200 - 5th Avenue, South
Lethbridge, Alberta
T1J 4C7
Phone: 381-5243
FAX: 381-5734

COPYRIGHT ACKNOWLEDGEMENTS

Alberta Education wishes to thank the following authors, publishers, and agents for granting permission to include copyrighted materials:

Alberta Consumer and Corporate Affairs for the excerpts from Moving Out, 1988.

Alberta Consumer and Corporate Affairs for the excerpts from Taking Charge of Your Money, 1988.

Alberta Consumer and Corporate Affairs for the excerpts from TIPS.

Alberta Consumer and Corporate Affairs for the excerpts from 2000 A.D. A Guide to Financial Awareness, 1989.

Alberta Education for the excerpts from Junior High Mathematics Teacher Resource Manual, 1988.

Alberta Education for the excerpts from Let Problem Solving be the Focus of the 1980's, 1983.

Alberta Education for the excerpts from Problem Solving Challenge for Mathematics, 1985.

Alberta Education for the excerpts from Problem Solving in Mathematics: Focus for the Future, 1987.

Boston, Allyn and Bacon for excerpts from Strategies for Teaching Students with Learning and Behaviour Problems, 1988, by Dr. C. Bos and S. Baughn.

Canadian Bankers' Association for excerpts from Credit Wise, 1983.

Division for Learning Disabilities, Learning Disabilities Focus, for material adapted from "Alternatives in the Assessment of the Learning Disabled Adolescent: A Learning Strategies Approach", by Judith Wiener, 1986.

Kitchener-Waterloo Record Ltd. for material adapted from That Figures, 1981, by Marilyn Hintz and Jerry Ziegler.

Mulcahy, Marlo and Peat for excerpts from SPELT: A Strategies Program for Effective Learning/Thinking, 1987, by permission of the authors.

National Council of Teachers of Mathematics for the excerpts from The Arithmetic Teacher, 1985 and 1988.

National Council of Teachers of Mathematics for the excerpts from Curriculum and Evaluation Standards for School Mathematics, Working Draft, 1987.

National Council of Teachers of Mathematics for the excerpts from How to Evaluate Progress in Problem Solving, 1987, by Randall Charles et al.

National Council of Teachers of Mathematics for the excerpts from The Mathematics Teacher, 1989.

END

U.S. Dept. of Education

Office of Education
Research and
Improvement (OERI)

ERIC

Date Filmed

March 29, 1991